OPTIMIZATION OF A PRESSURIZED WATER REACTOR CORE

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MASTER

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ABSTRACT

A parametric study of pressurized light water reactor cores was undertaken, in which both heat transfer and nuclear influences of the basic parameters on the quantity and required enrichment of the uranium fuel were to be considered.

Methods were developed for the following:

1. Computation of the minimum core size for a specified power output on the basis of heat transfer characteristics of the unit cell parameters.
2. Computation of the critical mass of U-235 required for a given reactor by an adaptation of a digital computer program written by J. R. Powell. This program, written for Whirlwind, is a Fourier solution for critical mass of a highly enriched, homogeneous, heavy water moderated reactor; methods were developed to adapt it to the low enrichment, heterogeneous, light water case, and to compute the necessary inputs.
3. Computation of the factor by which the critical enrichment of a given reactor would have to be increased in order to attain a specified core life.

A correlation was obtained between the adjusted Powell Fourier program and experimental data; the methods selected for calculation of resonance escape probability, fast fission factor, and disadvantage factor for use in the program were also confirmed by comparison of results with experimental data.

Ranges of parametric variation were chosen as follows:

1. Fuel elements to be rods, 0.2, 0.4, and 0.6 inches in diameter.
2. Ratio of volume of moderator to volume of fuel to be 1.5, 2.0, 3.0, 4.0, 5.0, and 6.0.
3. Cladding to be of stainless steel and of zirconium, 0.015 inches thick.
4. Fuel to be uranium oxide, UO_2; quantity and enrichment to be determined.
5. Cores to be right circular cylinders not to exceed 13 feet in diameter and core heights to be 4, 6, and 8 feet.
6. Cores to be surrounded by an effectively infinite water reflector.
7. Maximum fuel temperature: 4800°F.
   Coolant pressure: 2000 psia. Inlet temperature: 487°F
   Maximum temperature: 628°F
   Average reactor temperature: 508°F
8. Thermal power output: 400 megawatts.

For each case, an appropriate core diameter and total quantity of uranium were computed from the heat transfer characteristics associated with the parameters. With this, resonance escape probability, fast fission factor, and cross sections modified for disadvantage factor, the machine calculation was entered. The output of the machine calculation was critical

35765

mass of U-235; this was compared with the total quantity of uranium previously computed to give the required critical enrichment.

Analysis of the uranium quantity and enrichment data thus obtained indicated that:

1. Total quantity of uranium required, a function of the heat transfer characteristics of the unit cell, is approximately proportional to the square of the rod diameter; the allowable heat generation rate per unit length of the central rod is very nearly the same regardless of the rod diameter, and hence the number of rods required is very nearly the same for all rod diameters at a given height.

2. Critical enrichment is principally a function of the unit cell parameters and is nearly independent of the core size and shape between diameter to height of core ratios of 0.8 and 3.

3. For each rod diameter and cladding material an optimum moderator to fuel volume ratio exists at which the critical enrichment is a minimum. For the diameters tested, these were as follows:

Rod Diameter	Optimum V_1/V_o	
	Stainless Steel clad	Zirconium clad
0.6 inch	3.0	3.0
0.4 inch	3.75	4.0
0.2 inch	4.5	4.5

4. An optimum rod diameter at which minimum critical enrichment is lowest exists for rods clad with each cladding material. For zirconium clad rods, this occurs at about 0.45 inches rod diameter for stainless steel clad, at slightly over 0.6 inches.

Rod Diameter	Minimum critical enrichment	
	Stainless Steel clad	Zirconium clad
0.6 inch	.99	.89
0.4 inch	1.12	.85
0.2 inch	1.5	1.13

5. Critical enrichments are lower for zirconium clad rods than for stainless steel clad, of the order of 1% enrichment compared to the order of 2%.

6. Optimum critical enrichment does not necessarily indicate the optimum core; other factors which must be considered are total quantity of uranium, cost data for uranium, cladding materials, and fabrication of fuel elements and pressure vessels, and the additional enrichment necessary to attain a specified core life.

Cambridge 39, Mass.

20 May 1957

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Mass.

Dear Professor Hamilton:

We hereby submit our thesis entitled "Optimization of a Pressurized Water Reactor Core" in partial fulfillment for the degree of Master of Science in Naval Architecture and Marine Engineering and for the degree of Naval Engineer.

ACKNOWLEDGEMENTS

The authors wish to express their appreciation to Professor T. J. Thompson for so liberally giving his time and for his guidance and encouragement throughout the preparation of this thesis.

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TABLE OF CONTENTS

Page
Table of Contents ii
Nomenclature iii
I. Introduction 1
II. Procedure 24
III. Results 32
IV. Discussion of Results 41
V. Conclusions 58
VI. Appendices
  A. The Disadvantage Factor 60
  B. Method of Adjusting Homogeneous Program Constants to Obtain Critical Mass of a Heterogeneous Reactor 70
  C. Adjustments to Nuclear Constants for Powell Fourier Program 72
  D. Computation for Fast Fission Factor and Resonance Escape Probability 76
  E. Powell Cylindrical Fourier Program Inputs and Outputs 83
  F. Nuclear Data 95
  G. Computation of Core Radius 104
  H. Correlation of the Powell Fourier Program with Experiment 113
  J. Computation of the Additional Enrichment Required to Attain Specified Core Life 119
  K. Bibliography 129

NOMENCLATURE

A. General

Symbol	Description	Dimensions
A	Atomic weight	
A	Coolant channel cross sectional area	ft^2
a	Rod heat transfer area	ft^2
B^2	Buckling-a measure of the bending of the neutron flux at a point in a critical reactor	cm^{-2}
c	Coolant specific heat	BTU/lb-°F
D	Diffusion coefficient-the proportionality factor between neutron current and neutron flux gradient	
E	Energy	Mev
E	Dimensionless coefficient-related to the computation of the disadvantage factor. A function of κ and R. See Appendix A.	
F	"	
F	Hot channel factor-heat transfer safety factor	
f	Thermal utilization-the proportion of thermal neutrons absorbed in fuel to the total absorbed	
H	Core height	cm
h	Film heat transfer coefficient	BTU/hr-ft^2-°F
I	Modified Bessel function of the first kind	
J	Bessel function of the first kind	
K	Modified Bessel function of the second kind	

k_∞	Infinite multiplication factor-the ratio of the average number of neutrons produced in each generation to the average number of corresponding neutrons absorbed	
k	Thermal conductivity	BTU/hr-ft-°F
L^2	(Diffusion length)^2-1/6th of the mean square distance traveled by a thermal neutron before being absorbed	cm^2
M^2	Migration area-1/6th of the mean square distance from birth of fission neutron to capture of a thermal neutron	cm^2
N	$\frac{6.025 \times 10^{23}}{A}\rho$	atoms/ cm^3
P	Dimensionless coefficient-function related to temperature differences in heat transfer. See Appendix G.	
p	Resonance escape probability- the probability that a neutron will not be absorbed by U-238 in the resonance energy region	
Q	Dimensionless coefficient-function related to heat transfer resistance	
q	Slowing down density-rate at which neutrons reach thermal energy per unit volume	neutrons/ cm^3-sec
q''	Heat transfer rate. See Appendix G.	BTU/hr-ft^2
R,r	Radius	
t	Temperature	°F
U	Total heat transfer coefficient	BTU/hr-ft^2-°F
V	Volume	cm^3
x	Ratio of volume of uranium + clad to water in a unit cell	
ε	Fast fission factor-the proportion of total fissions at both fast and thermal energies in both U-235 and U-238 to the number of thermal fissions in U-235	

κ	Reciprocal diffusion length- $= 1/L$	cm^{-1}
λ	Reflector savings-the decrease in critical size of a reactor due to a reflector	cm
ρ	Density	gm/cm^3
σ_a	Microscopic absorption cross section-the probability that one atom of material will absorb a neutron	barns
σ_f	Microscopic fission cross section-the probability that one atom of material will be fissioned by a neutron	barns
σ_s	Microscopic scattering cross section-the probability that one atom of material will scatter a neutron	barns
σ_{tr}	Microscopic transport cross section-the product of σ_s and $(1-\bar{\mu}_0)$	barns
Σ_a	Macroscopic absorption cross section-the product of the number of atoms of material per unit volume and σ_a for that material	cm^{-1}
Σ_f	Macroscopic fission cross section-the product of the number of atoms of material per unit volume and σ_f for that material	cm^{-1}
Σ_{tr}	Macroscopic transport cross section-the product of the number of atoms of material per unit volume and σ_{tr} for that material	cm^{-1}
τ	Age-1/6th of the mean square distance traveled by a neutron in being slowed down from fission to thermal energy	cm^2
ν	Average number of fast neutrons released per slow neutron fission	
ϕ	Neutron flux-the product of the number of neutrons per unit volume and their velocity	neutrons/ cm^2-sec

B. Specific

Symbol	Description
a_o	rod heat transfer area, midpoint to end of core
a_f	rod heat transfer area midpoint extended through reflector savings
A_u	cross sectional area of unit cell
F_q	heat generation rate hot channel factor
$F_{\Delta T}$	temperature rise hot channel factor
F_θ	heat transfer coefficient hot channel factor
k_c	clad thermal conductivity
k_f	fuel thermal conductivity
P_t	reactor thermal power
$\overline{q}$	average power developed per rod averaged over core
q_o	average power developed in a rod averaged longitudinally only
q''_m	heat transfer rate at midpoint of the central rod
q''_{mBO}	heat transfer rate at midpoint of the central rod required to produce burnout of clad
q''_{mLB}	heat transfer rate at midpoint of the central rod required to produce local boiling at outlet
R_o	radius of fuel in a unit cell or to inside of cladding
R_1	outside radius of unit cell
R_c	radius to outside of cladding
R	core radius
R'	core radius plus reflector savings

Δt_c	temperature rise of coolant in core from inlet to maximum temperature
Δt_f	temperature rise from coolant inlet to centerline of fuel rod
Δt_{wi}	temperature rise from coolant inlet to cladding-fuel interface
Δt_{wo}	temperature rise from coolant inlet to outside of cladding
U_c	effective heat transfer coefficient for bond, clad, and scale
V_o	volume of fuel in unit cell
V_1	volume of clad + moderator in unit cell
V_c	volume of clad in unit cell
V_{w,H_2O}	volume of water in unit cell
V_u	volume of uranium in unit cell
V_s	volume of structure and clad in unit cell
χ_o	fuel reciprocal diffusion length
χ_1	moderator reciprocal diffusion length
Σ_{ao}	fuel macroscopic absorption cross section
Σ_{a1}	moderator macroscopic absorption cross section
Σ_{otr}	fuel macroscopic transport cross section
Σ_{1tr}	moderator macroscopic transport cross section
$\bar{\Sigma}_1$	slowing down cross section
τ_1	Fermi age
τ_2	diffusion age
$\bar{\Phi}_o$	volume average slow neutron flux in fel
$\bar{\Phi}_1$	volume average slow neutron flux in moderator
$\bar{\Phi}_c$	volume average slow neutron flux in clad
$\bar{\Phi}_1/\bar{\Phi}_o$	disadvantage factor

I.

INTRODUCTION

The category of nuclear reactors known as pressurized water reactors has emerged from the multitude of possible types of power producing reactors as one of the most promising from a feasibility viewpoint. It appears that over the next several years it will continue to be an important type both for ship propulsion and for shore power station use, and hence a study of the effects of varying some of the basic core parameters was considered to be a fruitful undertaking.

A pressurized water reactor is distinguished by the following characteristics:

1. Water is used as both the coolant and the moderator.
2. It is heterogeneous; that is, it is made up of segregated regions of fuel and coolant-moderator, separated by a corrosion resistant "cladding."
3. The water is kept under high pressure to prevent boiling.
4. The great majority of fissions is produced by neutrons at thermal energy, that is, neutrons which are in thermal equilibrium with the atoms of the surrounding medium.

The first full scale power reactor to operate, the prototype of the propulsion reactor in the U. S. submarine Nautilus, was of this type. Since then, in addition to the successful plant actually installed in the Nautilus, the pressurized water concept has been accepted as standard for U. S. submarines and thirteen other boats are building, in

the design state, or authorized. Aircraft carrier and cruiser plants are also being planned. The pressurized water reactor has also taken a leading position in the U. S. for shore power station use: the first civilian power reactor to operate will be the PWR at Shippingport, Pennsylvania, and among the ones to follow it will be the Yankee Atomic Electric Company's design, to be built at Rowe, Massachusetts. Similar reactors are being built in other countries; a list of those now planned follows:[1]

	Fuel	Heat Power	Critical Date
Army Package Power Reactor	UO_2	10 mw	Early 1957
Belgian Thermal Reactor	U	260	1957
Dominican Republic Reactor	UO_2	43	
Consolidated Edison Reactor	U	500	July 1959
University of Florida Reactor	U	10	December 1959

The uranium fuel in all these designs is formed into elements, such as plates or rods, which are small compared to the size of the reactor as a whole but very large compared to atomic distances and of the same order of magnitude as the mean free path of a neutron in uranium, about half an inch. These elements are regularly spaced in a vessel through which water can flow, filling the spaces between the elements. Each element with its share of the surrounding water can be thought of as a cell, and the whole reactor as an aggregate of such cells, or a cell lattice. Some of the nuclear and thermal properties of the reactor are properties of the cell parameters

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[1] "Nuclear Reactor Data Handbook No. 2", The Raytheon Mfg. Co.

while some are properties of the lattice. The properties of basic interest are:[1]

1. Nuclear properties of the unit cell.
   a. The infinite multiplication factor[2], k_{oo}. In order to maintain steady state operation, the number of neutrons in successive generations must be conserved. That is:

$$\text{Production} = \text{Absorption} + \text{Leakage}$$

k_{oo} is the number of thermal neutrons produced per thermal neutron absorbed if no leakage takes place. It can be thought of as the factor relating the numbers of neutrons in successive generations in a medium consisting of an infinite repetition of the unit cell. It is generally recognized as the product of four factors, which, with the energy cycle followed by the neutrons, are explained below.

η : The number of fast neutrons produced per thermal neutron absorbed in the fuel. High speed neutrons with a mean energy of about 1 mev are produced by absorption of thermal neutrons, those which have been so slowed down by

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[1] These properties are described in more detail in various references, and, in some cases, in the Appendices to this thesis. These references are cited in each case.

[2] See S. Glasstone and M.C. Edlund, "The Elements of Nuclear Reactor Theory", Van Nostrand (1952)(hereafter cited as "Glasstone and Edlund"), Section 4.57 ff.

successive collisons that their energies depend only on the temperature of the surrounding medium. Not all those absorbed in the fuel produce fissions; those that do, produce on the average 2.46 fast neutrons and about 192 mev of energy.

ϵ : *The fast fission factor*. Neutrons with energies greater than about 1 mev are capable of producing a small number of fissions in U-238. ϵ is the ratio of the total rate of neutron production to the production rate from thermal fission of U-235.

p : *The resonance escape probability*. U-238 has a high probability of absorption without fission for neutrons in the "resonance" energy range, from about 10 to 100 ev. p is the probability that a fission neutron in slowing down, will not be so captured.

f : *The thermal utilization* is the proportion of all the thermal neutrons absorbed which are absorbed in the fuel.

b. The disadvantage factor,[1] $\bar{\Phi}_1/\bar{\Phi}_0$. Thermal neutrons are produced in the moderator region as fast neutrons resulting from fissions in

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[1] See Glasstone and Edlund, 9.48, and Appendix A. Also see A. L. Kaplan, "Theoretical Studies of Neutron Flux Distribution Expected in the MIT Nuclear Research Reactor", MIT S.M. thesis, 1955.

the fuel region are slowed down by elastic collisions with the hydrogen nuclei.

The neutron flux,[1] ϕ, expresses the length of neutron paths covered per unit time per unit volume. This depends both on the number of neutrons and on their velocity. This is shown in the units of flux:

$$\text{neutrons} \times \frac{\text{cm}}{\text{sec.}} \times \frac{1}{\text{cm}^3} = \frac{1}{\text{cm}^2\text{-sec}}$$

The number of interactions taking place per unit time and volume is the product of the length of neutron paths and the probability of interaction per unit path length, Σ.

$$\frac{\text{no. of interactions}}{\text{unit time-unit volume}} = \Sigma\phi$$

Σ in turn consists of the number of nuclei per unit volume, N, times the reaction probability per nucleus, σ. Thus,

$$\frac{\text{no. of interactions}}{\text{unit time-unit volume}} = \Sigma\phi = N\sigma\phi$$

The fast flux produced by fissions in the fuel is slowed down in the moderator, producing thermal flux. As the thermal flux thus produced reenters the fuel, it is continuously reduced as the center of the fuel element is approached since Σ for absorption is greater in the fuel than in the moderator and since essentially no slowing down takes place in the fuel to

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[1] See Glasstone and Edlund, 3.49.

replenish the supply of thermal neutrons. The general pattern of thermal flux in a cell is shown in Figure 1.

The fuel near the center of the element, thus, meets a lower flux than the average over the whole reactor, with a consequent reduction of reaction probability below that which would be exhibited by a homogeneous mixture of the same materials in the same average thermal flux. The ratio of the average thermal neutron flux in the moderator to that in the fuel is the disadvantage factor.

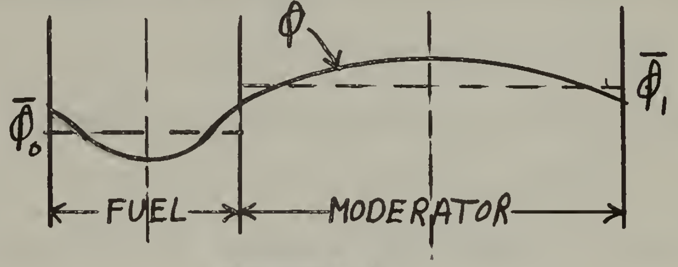

Figure 1

c. Fermi Age, τ.[1] Since the slowing down process is a complex one, involving fission neutrons with a distribution of initial energies taking part in a large number of elastic collisions in which their energy loss varies (for a hydrogen containing medium) between zero and 100%, it is represented by an experimentally determined number which is characteristic of the

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[1] See Glasstone and Edlund, 6.117 to 6.145, and Appendix F.

proportions of the various materials in the reactor; since these materials and proportions are repeated in all the cells, it can thus be considered a property of the cell. Physically, the Fermi age is one sixth of the average squared crow-flight distance travelled by a neutron during the process of slowing down from fission to thermal energy.

2. Nuclear properties of the reactor as a whole.
   a. Flux distribution,[1] $\phi(r)$. Due to the effects of leakage and changes of composition across the lattice, the flux varies over each dimension of the reactor, being high near the center and in regions of high multiplication factor, and low near the edges and in regions of low multiplication factor. It drops to zero a small distance outside the physical boundaries of the reactor at the "extrapolated boundary."[2] The functions which express these patterns are as follows:

Reactor Shape	Dimension	Function
Cube	parallel to an edge of length H	$\cos\left(\frac{\pi}{2}\frac{x}{H/2}\right)$
Right circular cylinder	radial (radius = R)	$J_0\left(\frac{2.405r}{R}\right)$
	axial (height = H)	$\cos\left(\frac{\pi}{2}\frac{x}{H/2}\right)$
Sphere	radial (radius = R)	$\frac{\pi r}{R}\ \sin\left(\frac{\pi r}{R}\right)$

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[1] See Glasstone and Edlund, 7.36 to 7.61
[2] See Glasstone and Edlund, 5.40

A reflector is frequently provided to reflect neutrons back into the core which would otherwise leak from it. When this is done, the average flux in the active core is increased, as shown in Figure 2, and the flux pattern is extended to the reflector boundary, c. The flux in the reflector has a different curvature from that in the core; but the core flux can be approximated by extending the point at which its characteristic function becomes zero by a distance such as ab, called the reflector savings.[1]

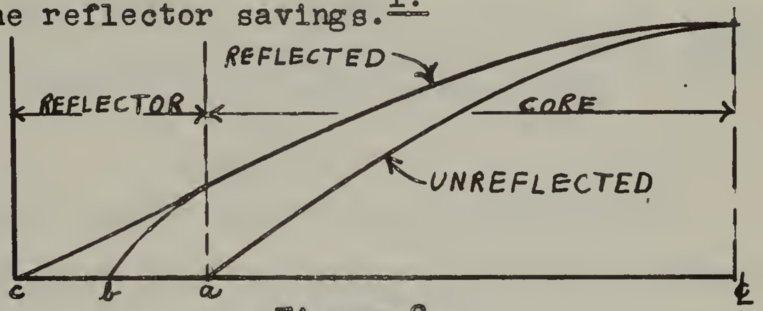

Figure 2

b. Buckling,[2] B^2, is a measure of the amount of curvature of the neutron flux pattern. The buckling which is required to make a given reactor critical can be found from the shape and dimensions of the reactor, and that which can be obtained by a given combination of materials can be found from the properties of the materials. The condition for criticality

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[1] See Glasstone and Edlund, 8.25, and Appendix F.
[2] See Glasstone and Edlund, 7.20 to 7.60.

of the reactor is that these two bucklings be equal. Buckling can also be used to find the proportion of neutron leakage from a given reactor. The probability of non-leakage during the slowing down process is $e^{-B^2\tau}$, and the ratio of thermal leakage to thermal absorption is L^2B^2, where L is the thermal diffusion length.[1]

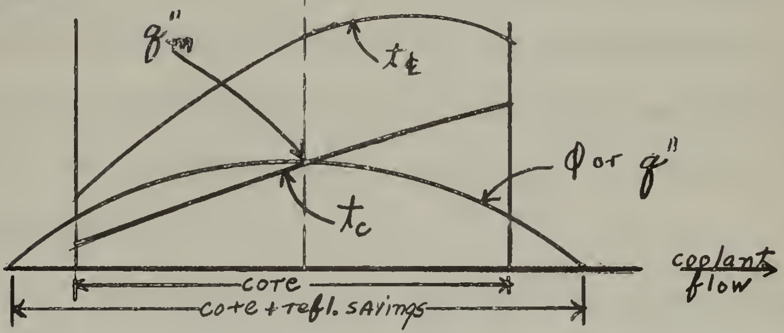

Figure 3

3. Thermal properties of the unit cell.

   a. Maximum heat generation rate[2]; q''_m. The rate at which heat can be generated in a fuel element is limited by its heat transfer properties. Along an element, the heat generation rate is proportional to the fission rate which is proportional to the thermal neutron flux, which will vary from zero at the ends of the reflector savings to a maximum halfway between them. For equal reflector savings at each

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[1] See Glasstone and Edlund, 5.62 to 5.91

[2] See W. M. Rohsenow, J. Lewins, and J. P. Barger, "Steady State Temperature Distribution in a Nuclear Reactor with End and Center Fed Coolant", NT-269, MIT, hereafter cited as Rohsenow, Lewins, and Barger, pages 1-4. Also see Appendix G.

end of the elements, this maximum will also be at the geometrical center of the active core. The coolant flows along the element and its temperature, t_c, rises as it does so to a maximum at the outlet, while the temperature at the element centerline, $t_{℄}$, rises to a maximum at a point between the middle of the element's length and the outlet, then drops off due to the reduction in the heat generation rate. The rate at which heat can be generated in a given element is limited by:

(1) the maximum fuel centerline temperature must be maintained below the melting point of the fuel material, and, for uranium metal fuel, below the phase change at 660 ^{o}F.

(2) the maximum coolant temperature must be less than the boiling point of the coolant. Boiling will change the water density, changing the density of hydrogen atoms in the moderator and thereby the slowing down characteristics and the reactivity of the reactor. In addition, boiling will reduce the heat transfer rate away from the element, leading to a temperature rise which may

be sufficient to melt it.

(3) The temperature of the cladding will follow a pattern between those of the element centerline and the coolant. This must also be maintained below the melting point of the cladding.

To improve the thermal efficiency of the plant, it is desirable to make the coolant outlet temperature as high as possible. Since this is limited by the boiling point of the coolant, the allowable temperature may be increased by increasing the water pressure. As can be seen from the curve of Figure 4[1] pressures greater than 2000 psia give very little gain in temperature; moreover, the critical point of water, at which the distinction between the liquid and vapor phases is lost, is at 3206.2°F, which places an absolute upper limit on the pressure. A more practical limit is imposed by the difficulties of manufacturing pressure vessels of the required sizes for pressures greater than around 2000 psia.

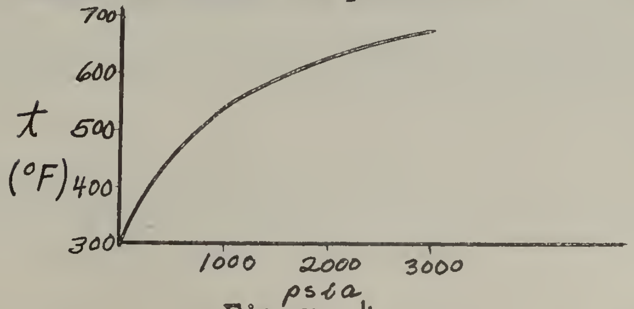

Figure 4

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[1] J. H. Keenan and F. G. Keyes, "Thermodynamic Properties of Steam", Wiley, (1936), page 39.

4. Thermal properties of the whole reactor.

   a. Overall heat generation rate.[1] The heat generation rate in each element will be proportional to its q_m''', which, in turn, is proportional to the flux distribution in a plane perpendicular to the axis of the element. Thus the maximum heat generation rate allowed for an element can exist only in one element, at the point of maximum flux, and all other elements will operate at heat generation rates less than the maximum. The ratio between the maximum and average heat generation rates and the maximum and average thermal neutron fluxes will be the same.

   b. Temperatures, t. The coolant inlet temperature is determined by the power system outside the reactor; the coolant outlet temperature is the temperature of the mixed outlets of all the channels. The average reactor and coolant temperatures can be found from the variations in point temperatures through the core.

A uranium fuelled, light water cooled and moderated, pressurized water reactor can be described by several basic parameters:

1. Describing the unit cell:
   a. Element shape and size.

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[1] See Appendix G.

   b. Volume ratio of moderator to fuel.
   c. Material and thickness of cladding.
   d. Fuel material and degree of enrichment in U-235.
2. Describing the reactor as a whole:
   a. Core shape and dimensions.
   b. Reflector dimensions.
   c. Temperature limitations.
   d. Power output.

Changes in these parameters act and interact in complex ways to affect the performance of the reactor. The number of parameters involved and the ranges of possible variation make experimental determination of the optimum combination for given requirements a long, tedious, and expensive process.

Exponential experiments[1] have been performed by H. J. Kouts at the Brookhaven National Laboratory on a number of the combinations[2] and some of these have been continued to criticality by S. Krasik and A. Radkowsky (of the U. S. Atomic Energy Commission) at the Westinghouse Electric Company's Bettis Plant.[3] Both these series involved uranium metal fuel elements;

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[1] See Glasstone and Edlund, 9.90 to 9.111.

[2] Herbert Kouts, "Intracell Flux Traverses and Thermal Utilizations for 1.15% Enriched Uranium Rods in Ordinary Water", Brookhaven National Laboratory Report No. 1987, August 11, 1954. H. J. Kouts, J. Chernick, and I. Kaplan, "Exponential Experiments on Light Water Moderated 1 per cent U-235 Lattices", Brookhaven National Laboratory Report No. 2094, November 28, 1952.

[3] S. Krasik and A. Radkowsky, "Pressurized Water Reactor Critical Experiments", paper no. 601, presented at the Geneva Conference on the Peaceful Uses of Atomic Energy, 1955.

a similar, though shorter, series of critical experiments on uranium oxide elements has been made by J. R. Brown, A. Z. Krantz, and others at Bettis.[1]

These data are not by themselves sufficient to select an optimum configuration for a given set of requirements, but they are sufficient to provide experimental verification of calculations.

The experimental results include critical bucklings, and, in those continued to criticality, critical number of rods, disadvantage factors, fast fission factors, resonance escape probabilities, and thermal utilizations for a series of rod diameters and volume ratios at a limited number of enrichments between 1 and 1.3%. In addition, Kouts' work includes some extremely valuable plots of relative neutron flux along various lines through the cells. All the fuel was in rod form and clad with aluminum, with active core heights of 48 inches. All cores were surrounded by effectively infinite water reflectors.

For the purpose of theoretical investigation of a number of different configurations, several mathematical models are available, varying in both complexity and accuracy. The

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[1] J. R. Brown, B. H. Noordoff, F. S. Frantz, J. J. Volpe, and D. R. Harris, "Measurement on Low Enrichments Light Water Moderated Critical Assemblies of U and UO_2", paper no. XVII-5 presented at the Chicago Meeting of the American Nuclear Society, 1956.
A. Z. Krantz, G. O. Smith, D. Klein, and W. Baer, "f, p, and Measurements on Slightly Enriched Light Water Moderated UO_2 Lattices", paper no. XVII-6 presented at the Chicago Meeting of the American Nuclear Society, 1956.

simplest of these, the "one group" equation,[1] assumes that all neutrons are at thermal energy. The critical condition is

$$1 = \frac{k_\infty}{1 + B^2L^2}$$

This is relatively simple mathematically, but poor in accuracy. Somewhat better in accuracy and not much more difficult in computation is the "modified one group" theory,[2] which includes a factor for leakage during the slowing down process. The critical condition may be expressed as

$$1 = \frac{k_\infty \; e^{-B^2\tau}}{1 + B^2L^2}$$

the Fermi Age variant, or

$$1 = \frac{k_\infty}{1 + B^2M^2}$$

where M^2, the migration area,[3] is $(L^2 + \tau)$, for a large reactor.

More complex is the "two group" theory,[4] which assumes a fast and a thermal group of neutrons with average reactor properties for each. Four differential equations are formed, expressing fast and thermal flux in the core and the reflector

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[1] See Glasstone and Edlund, 8.15 to 8.37.
[2] See Glasstone and Edlund, 7.20 to 7.27, and 7.62 to 7.64
[3] See Glasstone and Edlund, 7.63
[4] See Glasstone and Edlund, 8.38 to 8.58

$$D_{1c}\nabla^2\phi_{1c} - \Sigma_{1c}\phi_{1c} + k_\infty\Sigma_{2c}\phi_{2c} = 0$$

$$D_{2c}\nabla^2\phi_{2c} - \Sigma_{2c}\phi_{2c} + \Sigma_{1c}\phi_{1c} = 0$$

$$D_{1r}\nabla^2\phi_{1r} - \Sigma_{1r}\phi_{1r} = 0$$

$$D_{2r}\nabla^2\phi_{2r} - \Sigma_{2r}\phi_{2r} + \Sigma_{1r}\phi_{1r} = 0$$

Where the subscripts 1 and 2 refer to the fast and thermal fluxes respectively and the subscripts c and r refer to the core and reflector respectively.

These are solved by means of boundary conditions which require both fluxes to be continuous across the core-reflector boundaries and both fluxes to go to zero at the outer reflector extrapolated boundary. These solutions lead to a critical determinant,

$$\begin{vmatrix} X & Y & -Z_1 & 0 \\ S_1X & S_2Y & -S_3Z_1 & -Z_2 \\ D_{1c}X' & D_{1c}Y' & -D_{1r}Z_1 & 0 \\ S_1D_{2c}X' & S_2D_{2c}Y' & -S_3D_{2r}Z_1' & -D_{2r}Z_2' \end{vmatrix} = 0$$

where, for a circular cylinder with an infinite radial reflector,

$$x = J_0(\mu r) \qquad y = I_0(\nu r)$$

$$S_1 = \frac{D_{1c}}{\tau_c D_{2c}} \times \frac{P}{\frac{1}{L_{2c}^2} + \mu^2} \qquad S_2 = \frac{D_{1c}}{\tau_c D_{2c}} \times \frac{P}{\frac{1}{L_{2c}^2} - \nu^2}$$

$$Z_{\frac{1}{2}} = K_0(\kappa_{\frac{1}{2}} r) \qquad S_3 = \frac{\tau_r D_{1r}}{D_{2r}} \, \frac{1}{(\kappa_{2r}^2 - \kappa_{1r}^2)}$$

$$\mu^2 = \frac{1}{2}\left[-\left(\frac{1}{\tau_c} + \frac{1}{L_{2c}^2}\right) + \sqrt{\left(\frac{1}{\tau_c} + \frac{1}{L_{2c}^2}\right)^2 + \frac{4(k_\infty - 1)}{\tau_c\, L_{2c}^2}}\right]$$

$$-\nu^2 = \frac{1}{2}\left[-\left(\frac{1}{\tau_c} + \frac{1}{L_{2c}}\right) - \sqrt{\left(\frac{1}{\tau_c} + \frac{1}{L_{2c}^2}\right)^2 + \frac{4(k_\infty - 1)}{\tau_c\, L_{2c}^2}}\right]$$

D = diffusion coefficient[1]

τ = Fermi Age

Coefficients for other geometries are given in Glasstone and Edlund, sections 8.46 and 8.51.

This treatment is capable of considering only one reflector. But in practical reactors, reflectors are usually provided for all exposed faces. This situation can be handled with moderate accuracy by Hill's Approximation,[1.] in which the reactor is considered separately for each reflected dimension and the solutions superimposed. In this approximation, the corners are not properly considered, but the flux is generally low in those areas and the resulting error not excessive.

It is possible to reduce the labor involved in critical computations considerably by use of a digital computer. These calculations are particularly useful in an investigation such as this, where a large number of similar problems will have to be solved by identical methods, for once the program is written, it can be used to solve any number of cases.

Two such programs for Whirlwind, the high speed digital computer at MIT were available, both written by James R. Powell as part of his MIT thesis for the Sc.D. degree.[2.]

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1. Glasstone and Edlund 5.7 and 5.20 to 5.24
2. Powell, James R., "Nuclear Characteristics of Heavy Water Moderated, Enriched, Homogeneous Reactors", Sc.D. thesis, MIT, 1957

These programs were based on the assumptions of a heavy water moderator, highly enriched uranium, and a homogeneous core. One of these programs was a computer representation of the two group Hill's approximation method already discussed; the other was based on a Fourier treatment of the diffusion equations.

The theory of the Fourier treatment will be briefly outlinedhere.[1] A detailed description of the theory for the Fourier program is contained in Powell's thesis. The thermal diffusion equation states that the losses due to leakage and absorption must equal the number of neutrons slowing down to thermal energy for a critical reactor.

$$\underset{\text{Leakage}}{\frac{\nabla^2\phi}{D}} - \underset{\text{absorption}}{\frac{\Sigma_a\phi}{D}} + \underset{\text{production}}{\frac{q}{D}} = 0 \qquad (1)$$

Each of the terms in equation (1) can be expanded in a Fourier series with eigenfunctions obtained from the wave equation

$$\nabla^2 Y_i + B_i^2 Y_i = 0$$

to satisfy boundary conditions. This thermal flux expansion would take the form of

$$\phi = \sum_{i=0}^{\infty} C_i Y_i$$

$$\frac{\Sigma_a\phi}{D} = \sum_{i=0}^{\infty} G_i Y_i$$

$$\frac{q}{D} = \sum_{i=0}^{\infty} A_i Y_i$$

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[1] Based on J. R. Powell's Thesis, Section III.

Since absorption and slowing down vary across the reactor and depend on factors other than flux, two further expansions are required

$$\frac{k_\infty(r)\,\Sigma_a(r)\,\phi(r)}{D(r)} = \sum_{l=0}^{\infty} E_l Y_l(r)$$

Since $\Sigma_a(r)$ is not constant, but varies as a step function of (r) from region to region an additional equation is written relating the Σ_a with the regions. The coefficients of these expansions are then related to the Fourier coefficients of the thermal flux expansion. Using these relations, an infinite series of equations is obtained with one equation for each unknown Fourier coefficient. For non-trivial solutions the determinant of the coefficients must equal zero. This constitutes the critical equation. In the program the critical mass is found using a limiting process, by finding the mass of U-235 needed to make the determinant of the first two rows and columns vanish, then the determinant of the first three rows and columns vanish and noting that the mass approaches a limiting value as the number of rows and columns increases.

Powell points out two major advantages to the Fourier solution of diffusion equations. The first of these is the ability to represent the slowing down behavior of fast neutrons as a convolution of an age and diffusion kernel. The Fermi age kernel is expressed by

$$P_l = \exp(-\tau_1 B_l^2)$$

and the diffusion kernel is expressed by

$$P_2 = \frac{1}{1 + \tau_2 B_1^2}$$

By postulating that the neutrons followed Fermi age theory until they reached energy E_1 and then diffused away from the source until a collision reduced their energy to thermal energy, E_{th}, it was felt by J. Powell that effectively two fast groups of neutrons were being considered. The slowing down kernel was then represented as

$$P = P_1 P_2 = P_1 \left[(E_{th} - E_1), B_1^2 \right] P_2 \, (E_1, B_1^2)$$

$$= \frac{e^{-\tau_1 B_1^2}}{1 + \tau_2 B_1^2}$$

The relative size of τ_1 and τ_2 was determined by matching the observed distribution of slowing down density with the calculated distribution so that no definite energy boundary between fast groups is established.

The second major advantage of the Fourier treatment pointed out by Powell was the relative ease with which additional reflectors could be handled. As a result of this, the Powell program included the provision for two top and two bottom reflectors and for six radial regions, any one of which could be the core region. All cores in this thesis are analyzed or designed on the basis of two regions only, a core and an outer infinite water reflector on top, bottom and sides.

A choice of two methods of machine computation was thus offered: the Fourier treatment and the two group Hill's approximation. For several reasons, the Fourier program seemed preferable for the purposes of the thesis, and it was ultimately selected. These reasons were:

1. The simultaneous solution of two cases required for Hill's approximation makes the solution rather lengthy. Computer time required for this program is about four minutes compared to two minutes for the Fourier.
2. Comparisons made by Powell in his thesis of the calculated results by both methods against experimental data for highly enriched, heavy water moderated cores showed smaller errors in the critical masses calculated by the Fourier program. Errors in cylindrical geometry varied from + .08 to - 14% with the Fourier and from - 4.0 to - 20% with the two group Hill's approximation.
3. Use of a two group program requires inputs of fast group constants. These are not well known, and, since a variety of materials was to be used in the core, it appeared that the introduction of this added uncertainty would not be advantageous. The Fourier program, on the other hand, uses only thermal constants and a Fermi Age, all of which are known to fair accuracy.

Having the required program inputs, the data available for experimental checks, the practical problems of fabrication which would limit the choices in a real system in mind, the following ranges of parameter variation were selected:

1. In the unit cell.
    - a. Fuel elements to be rods, from 0.1 to 0.6 inches in diameter.
    - b. Volume ratios, moderator to fuel, from 1.5 to 6.0.
    - c. Cladding to be .015 inches thick, both stainless steel and zirconium to be considered.
    - d. Fuel to be uranium oxide, UO_2; quantity and enrichment to be determined.
2. For the whole reactor.
    - a. Cores to be right circular cylinders not to exceed 13 feet in diameter, and to vary between 4 and 8 feet in height.
    - b. Cores to be surrounded by an 18 centimeter thick water reflector both radial and top and bottom; this is effectively an infinite reflector.
    - c. Maximum temperature in fuel: 4800°F

      Coolant pressure: 2000 psia

      Coolant temperatures: Maximum: 628°F

      Inlet : 487°F

      Average: 508°F
    - d. Thermal power output: 400 megawatts.

In order to carry out the parametric study utilizing the machine calculation, a series of steps were undertaken:

1. A method was developed to convert the program from the high enrichment, homogeneous case with heavy water moderator and aluminum structure for which it was written, to the case which was to be investigated: low enrichment, heterogeneous, light water moderated, and stainless steel or zirconium structure.
2. A method was developed for designing a consistent series of cores for testing which reflects the influence of parameters to be varied both on the heat transfer and the nuclear characteristics of the core, and supplies the necessary data for entering the machine calculation.
3. A survey was made to collect best values of the necessary constants and basic data, and to select methods for the various preparatory calculations required.
4. Correlations were obtained, both for the preparatory calculations and for the computer results, with the experimental data available.

II.
PROCEDURE

The use of the Whirlwind program selected, which was written for a highly enriched, heavy water moderated, homogeneous reactor, for computations on a low enrichment, light water moderated, heterogeneous reactor involved a considerable amount of calculation prior to entering the machine computation.

The program assumes the existence of only three materials in the core:

1. Fuel, assumed to be U-235.
2. Moderator, assumed to be D_2O.
3. An absorber, assumed to be aluminum.

Appropriate values for the densities and atomic weights of these materials are built into the program. Further, the program assumes that the core region consists of a uniform mixture of moderator and absorber with a small amount of fuel uniformly distributed in the mixture. Among the computer inputs[1] are values of the microscopic absorption and transport cross sections of moderator and absorber, the volume ratio of absorber to moderator, and the microscopic absorption and fission cross sections of the fuel. The principal problem in utilization of the program was to compensate within these inputs for the effects of heterogeneity, the use of materials other than heavy water and aluminum as moderator and absorber, and the additional presence of relatively large quantities of U-238, for which the program makes no provision.

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[1] See Appendix E.

In principle, two reactors, one heterogeneous, the other homogeneous are equivalent if the average values of macroscopic absorption, fission, and transport cross sections for the entire reactors are equal. This is the basis of the derivations for the necessary adjustments to the cross sections as outlined in Appendix B. Each value of a thermal cross section is adjusted to have the same value as the effective volume average value of the corresponding cross section in the heterogeneous reactor weighted by the relative flux through the unit cell. For absorption in the fuel, as an example, this is expressed as follows:[1]

$$\bar{\Sigma}_a(25) = \frac{N(25)\sigma_a(25)\, V_0\, \bar{\Phi}_0}{V_0\, \bar{\Phi}_0 + V_1\, \bar{\Phi}_1} = N'(25)\sigma_a'(25) \tag{1}$$

$$\bar{\Sigma}_a(25) = N(25)\sigma_a(25)\, \frac{V_0}{V_0 + V_1 \frac{\bar{\Phi}_1}{\bar{\Phi}_0}} = N(25)\, \frac{V_0}{V_0 + V_1}\, \sigma_a'(25) \tag{2}$$

$$\sigma_a'(25) = \sigma_a(25)\, \frac{V_0 + V_1}{V_0 + V_1 \frac{\bar{\Phi}_1}{\bar{\Phi}_0}} \tag{3}$$

where N(25) and $\sigma_a(25)$ are the number of atoms per cubic centimeter of fuel and the microscopic cross section in the heterogeneous reactor and N'(25) and $\sigma_a'(25)$ are the number of atoms per cubic centimeter of core and the microscopic cross section in the equivalent homogeneous reactor, $\sigma_a'(25)$ is the appropriate computer input.

$\bar{\Phi}_1/\bar{\Phi}_0$ is the disadvantage factor, discussed in the Introduction. Methods of computing the disadvantage factor are discussed in Appendix A. The choice of method for use in this thesis was based on the results of A. L. Kaplan's M.I.T.

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[1] See Appendix B.

thesis,[1] in which he compared the accuracy of the disadvantage factor for the M.I.T. Reactor as computed from three increasingly sophisticated models. His conclusion was that the most complex of the three models offered only a three per cent increase in accuracy over the simplest, and that the added accuracy did not justify the added labor of computation. The model chosen, then, was the simplest of Kaplan's three; it assumes constant fast flux across the cell and no net thermal leakage from the cell.[2]

Since the program is based on pure U-235 as the fuel, it appeared that the best way to include the effects of U-238 was to consider it as part of the absorber, which then consisted of both U-238 and the cladding material. The process developed for adjusting the constants for this arrangement was as follows:

1. A macroscopic cross section was formed for the U-238 from N(28) in the fuel and $\sigma_a'(28)$, obtained by a disadvantage factor adjustment as in equation (3) above.
2. A macroscopic cross section was obtained for the cladding material. Since the cladding is in a region of the unit cell in which the flux is very nearly the average value, no disadvantage factor correction was included, and the macroscopic cross section consisted simply of the product N(clad) x σ_a(clad).

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[1] A. L. Kaplan, "Theoretical Studies of Neutron Flux Distribution Expected in the MIT Nuclear Research Reactor", MIT S.M. Thesis, 1955.

[2] Also discussed in Glasstone and Edlund, 9.47 to 9.66, and in Appendix A.

3. These cross sections were weighted by their respective volumes and averaged over the total volume of fuel and cladding.
4. Since the machine would compute a macroscopic absorber cross section on the basis of the density and atomic weight of aluminum, which were integral parts of the program, the average macroscopic cross section obtained in step (3) was divided by N(Al).

Mathematically, the process is described by the following equation:

$$\sigma(\text{absorber}) = \frac{N(28)\sigma'(28)\ V_o + N(\text{clad})\sigma(\text{clad})\ V(\text{clad})}{V_o + V(\text{clad})} \ \mathrm{x}$$

$$\frac{A_{Al}}{\rho_{Al} \ \mathrm{x}\ 6.025 \ \mathrm{x}\ 10^{23}} \qquad (4)$$

A step for spreading the absorber uniformly through the core volume is part of the computer program; a program input, "x", is the volume ratio of absorber to moderator. For computing "x", the volume of fuel rod plus cladding was used.

A further adjustment must be made for the presence of U-238 in the fuel rods. With highly enriched uranium, there is neither a fast fission effect nor resonance absorption, and the infinite multiplication factor, k_{oo} is simply

$$k_{oo} = \eta f$$

η for the fuel is a program input; f is computed by the machine. However, for low enrichments,

$$k_{oo} = \eta \varepsilon p f$$

In order to use the program for low enrichments, then, ε and p were separately calculated, as described in Appendix D, and the product was used as the program input "η".

An additional arbitrary factor of 1.1 was used in the program input "η" to bring computed results into agreement with experimental data. The derivation of this factor is discussed in Appendix H. Thus,

$$"\eta" = \eta(25)\,\varepsilon\, p \times 1.1$$

Similar adjustments were made for the substitution of light water at 508°F for the heavy water at room temperature on which the program is based. Here the adjustment process was as follows:

$$\sigma(\text{moderator}) = \frac{\rho_{H_2O}}{A_{H_2O}} \times \sigma_{H_2O} \times \frac{V_1 + V_0}{V_1 + V_0\,\Phi_0/\Phi_1} \times \frac{A_{D_2O}}{\rho_{D_2O}}$$

Appendix F lists all cross sections required to calculate both the cores for comparison with experiment and cores for our parametric study. Cross section information was obtained from BNL-325, Neutron Cross Sections, and modified for Maxwell-Boltzmann distribution and additionally for temperature where necessary. As pointed out in the Introduction, one of the advantages of the Fourier program is its ability to use both a Fermi age and a diffusion age and in this manner effectively treat the fast neutrons as two groups. A rather complete survey of the literature was made to determine whether any theoretical work had been done on defining age in this way for light water, and it appeared that it had not. The most authoritative experimental results were computed by Kouts, et.al. at Brookhaven

based on the Fermi Age theory. These results indicated that $\tau = 31\ \text{cm}^2$ for light water of density 1 gm/cm^3 for water to metal ratios from one to three. It was therefore decided to use this number as the Fermi age and to set the diffusion age input equal to zero thus eliminating this. The resulting calculation performed by the program is actually a one group continuous slowing down solution. Appendix F includes a fuller description of this literature survey.

In addition to the neutron characteristics of the material components of the reactor, Powell's program also requires the dimensions of the reactor as inputs. Since uniform reflectors and cylindrical geometry were assumed for all cases, the only dimensions required were core height and radius. A series of set core heights was assumed; for given height, power output, and cell geometry, the radius is determined by the heat transfer characteristics and temperature limitations of the materials and the overall flux distribution.

A method of computing core radius on the basis of cell geometry and materials for given power output and core height was developed from the functions derived by Rohsenow, Lewins, and Barger[1]. for temperature distributions in a core. This development, which is carried through in detail in Appendix G, assumes that the optimum configuration is that in which the temperature limitations for both bulk boiling in the water and melting of the fuel element are just met at the hottest points in these media. A third possible criterion might have been that the melting point of the cladding should just be reached;

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[1] Rohsenow, Lewins, and Barger, op.cit., and Appendix G

the temperature of the hottest point in the cladding was checked in each calculation and found to be well below the melting temperature; thus this was not a limiting factor.

This method offers considerable advantages in a study such as this. By using it, it is possible to design a series of cores of widely varying geometry all of which will be capable of developing the same amount of thermal power while just meeting an identical set of temperature limitations. This allows meaningful comparisons between the different combinations of parameters.

The process carried through on each of the cores considered was as follows:

1. Characteristics were selected from the ranges of parameter variations listed in the Introduction. These were: rod diameter, moderator to fuel volume ratio, cladding material (stainless steel or zirconium), and core height.
2. Using these characteristics, adjusted cross sections were computed.
3. Using the same characteristics, a core radius was computed; since this was the boundary of a uniform lattice of cells each of known geometry, the number of cells and hence the total amount of fuel was also known.
4. Using the results of steps (2) and (3), as input data, a Whirlwind computation using Powell's Fourier program was made to find the critical

mass of U-235 required.

5. The critical mass of U-235 was compared to the total mass of uranium, found in step (3), giving the required enrichment of the uranium for criticality in that core.
6. Cores were compared on the basis of amount and enrichment of uranium required.

It will be noted that enrichment is required in order to find the disadvantage factor, and therefore in order to compute the adjusted cross sections; it is also required to determine the amount of U-238 present and hence the U-238 contribution to the "absorber" cross sections. A solution can thus only be obtained by an iterative process in which an enrichment is assumed in step (2) and checked in step (5). However, since the range of enrichments involved is very small, from about one to three percent, and the solution converges very rapidly, only a few cores had to be run more than once, and none more than twice.

III.
RESULTS

TABLE I

Total Mass of Uranium (10^6 gm)

Stainless Steel and Zirconium

Rod Diameter	Core Height	V_w/V_u					
		1.5	2.0	3.0	4.0	5.0	6.0
0.1"	8'						
	6'					.96	
	4'			1.043	1.10	1.19	
0.2"	8'	4.37	4.57	4.66	4.76	4.88	4.92
	6'		4.66	4.85	4.92	4.96	5.05
	4'		4.89	5.02	5.13	5.17	5.21
0.4"	8'	19.8	20.2	20.5	20.7	20.7	
	6'	20.35	20.6	20.9	21.0	21.2	
	4'	20.9	21.1	21.3	21.4	21.8	
0.6"	8'	46.1	47.0	48.0	48.5	48.6	
	6'	47.0	47.6	48.5	49.0		
	4'	48.2	48.8	49.3	50.2		

TABLE II

Stainless Steel Clad

U-235 Critical Mass (10^4 gm)

Rod	Core	V_1/V_0					
Diameter	Height	1.5	2.0	3.0	4.0	5.0	6.0
0.1"	8'						
	6'					3.72	
	4'			6.44	4.25	3.60	
0.2"	8'	12.6	11.1	8.85	7.94	7.82	8.45
	6'		11.23	8.77	7.61	7.65	8.21
	4'		10.7	8.76	7.80	7.88	8.44
0.4"	8'	35.1	30.3	26.0	24.5	27.26	
	6'	34.6	29.9	25.0	24.1	27.13	
	4'	30.7	28.1	25.0	24.2	27.51	
0.6"	8'	55.4	51.6	50.1	53.2	59.9	
	6'	51.2	50.6	49.2	50.8		
	4'	51.5	49.7	48.8	52.22		

TABLE III

Zirconium Clad

$M_{25} \times 10^4$ gm

Rod Diameter	Core Height	V_1/V_o					
		1.5	2.0	3.0	4.0	5.0	6.0
0.1"	8'						
	6'					1.88	
	4'			2.89	2.04	1.80	
0.2"	8'	7.22	6.53	5.60	5.37	5.56	5.94
	6'		6.64	5.86	5.63	5.81	6.20
	4'		7.19	6.22	6.05	6.14	6.56
0.4"	8'	25.4	21.56	18.5	17.65	20.37	
	6'	26.06	22.46	19.1	18.67	21.86	
	4'	27.59	23.4	19.7	19.38	22.34	
0.6"	8'	43.06	42.8	43.0	44.95	51.83	
	6'	43.81	43.0	43.2	44.13		
	4'	44.81	44.1	44.02	45.11		

TABLE IV

Enrichment

Stainless Steel Clad

Rod Diameter	Core Height	V_1/V_0					
		1.5	2.0	3.0	4.0	5.0	6.0
0.1"	8'						
	6'					3.88	
	4'			5.62	3.75	3.02	
0.2"	8'	2.88	2.43	1.90	1.67	1.60	1.715
	6'		2.41	1.81	1.55	1.54	1.625
	4'		2.19	1.74	1.52	1.525	1.6
0.4"	8'	1.775	1.50	1.26	1.182	1.32	
	6'	1.70	1.45	1.20	1.15	1.28	
	4'	1.465	1.33	1.172	1.13	1.26	
0.6"	8'	1.20	1.10	1.045	1.10	1.23	
	6'	1.09	1.06	1.015	1.035		
	4'	1.07	1.02	0.99	1.04		

TABLE V

Enrichment

Zirconium Clad

Rod	Core	V_1/V_o					
Diameter	Height	1.5	2.0	3.0	4.0	5.0	6.0
0.2"	8'	1.65	1.425	1.20	1.13	1.14	1.205
	6'		1.43	1.21	1.145	1.17	1.225
	4'		1.47	1.24	1.18	1.185	1.26
0.4"	8'	1.28	1.07	.90	.85	.985	
	6'	1.285	1.09	.915	.89	1.03	
	4'	1.32	1.11	.925	.905	1.035	
0.6"	8'	.935	.91	.895	.927	1.065	
	6'	.932	.905	.890	.905		
	4'	.930	.905	.892	.900		

TABLE VI

Core Diameter to Height Ratio

Rod Diameter	Core Height	V_1/V_0					
		1.5	2.0	3.0	4.0	5.0	6.0
0.1"	8'						
	6'						
	4'			.60	.66	.754	
0.2"	8'	.332	.37	.422	.49	.54	.585
	6'		.577	.68	.765	.84	.917
	4'		1.078	1.26	1.41	1.56	1.70
0.4"	8'	.705	.778	.905	1.01	1.11	
	6'	1.11	1.21	1.41	1.57	1.74	
	4'	2.03	2.24	2.60	2.91	3.20	
0.6"	8'	1.10	1.18	1.38	1.55	1.70	
	6'	1.71	1.84	2.15	2.41		
	4'	3.15	3.55	3.95	4.45		

TABLE VII

k_∞

Stainless Steel

Rod Diameter	Core Height	V_1/V_e					
		1.5	2.0	3.0	4.0	5.0	6.0
0.1"	8'						
	6'						
	4'			1.16	1.15	1.12	
0.2"	8'	1.13	1.13	1.11	1.10	1.09	1.08
	6'		1.13	1.11	1.06	1.05	1.04
	4'		1.15	1.12	1.08	1.06	1.05
0.4"	8'	1.19	1.19	1.13	1.11	1.09	
	6'	1.08	1.08	1.10	1.09	1.07	
	4'	1.12	1.11	1.09	1.08	1.07	
0.6"	8'	1.11	1.11	1.09	1.08	1.07	
	6'	1.08	1.11	1.06	1.05		
	4'	1.07	1.09	1.06	1.05		

TABLE VIII

k_∞

Zirconium

Rod Diameter	Core Height	V_1/V_0					
		1.5	2.0	3.0	4.0	5.0	6.0
0.1"	8'						
	6'						
	4'			1.18	1.16	1.15	
0.2"	8'	1.14	1.15	1.13	1.11	1.10	1.09
	6'		1.14	1.13	1.12	1.11	1.09
	4'		1.17	1.17	1.15	1.13	1.10
0.4"	8'	1.20	1.18	1.10	1.06	1.05	
	6'	1.21	1.19	1.11	1.08	1.07	
	4'	1.22	1.19	1.10	1.08	1.07	
0.6"	8'	1.12	1.11	1.09	1.08	1.07	
	6'	1.13	1.11	1.10	1.07		
	4'	1.13	1.10	1.09	1.05		

IV.

DISCUSSION OF RESULTS

The results of the calculations for the series of 400 megawatt cores described in the Introduction are shown in the Results, and total uranium mass and enrichment are plotted in Figures 2, 3 and 4. Both the total mass of uranium and the required critical enrichment are shown, since the uranium cost is a function of both.

Total mass of uranium is a function solely of the heat transfer properties of the combination of parameters under consideration. It is a strong function of rod diameter and a weak function of core height and moderator to fuel volume ratio. For a given core height the allowable heat generation rate in the central rod is almost independent of rod diameter. This is due to the low thermal conductivity of the oxide fuel (0.95 Btu/hr.-ft.-^{o}F) compared to those of stainless steel (10.6) or zirconium (8.15). As a result, almost the entire temperature drop between the 4800^{o}F fuel centerline and the water at less than 628^{o}F is taken across the fuel region. Using the methods of Rohsenow, Lewins, and Barger,[1] a sample temperature distribution was computed for the point of maximum fuel centerline temperature in a 0.4 inch diameter rod. It is shown in Figure 1, in which the magnitude of the drop in the fuel, in this case 97% of the total drop, can be seen and compared with those in the clad and across the film.

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[1] Rohsenow, Lewins, and Barger, op. cit.

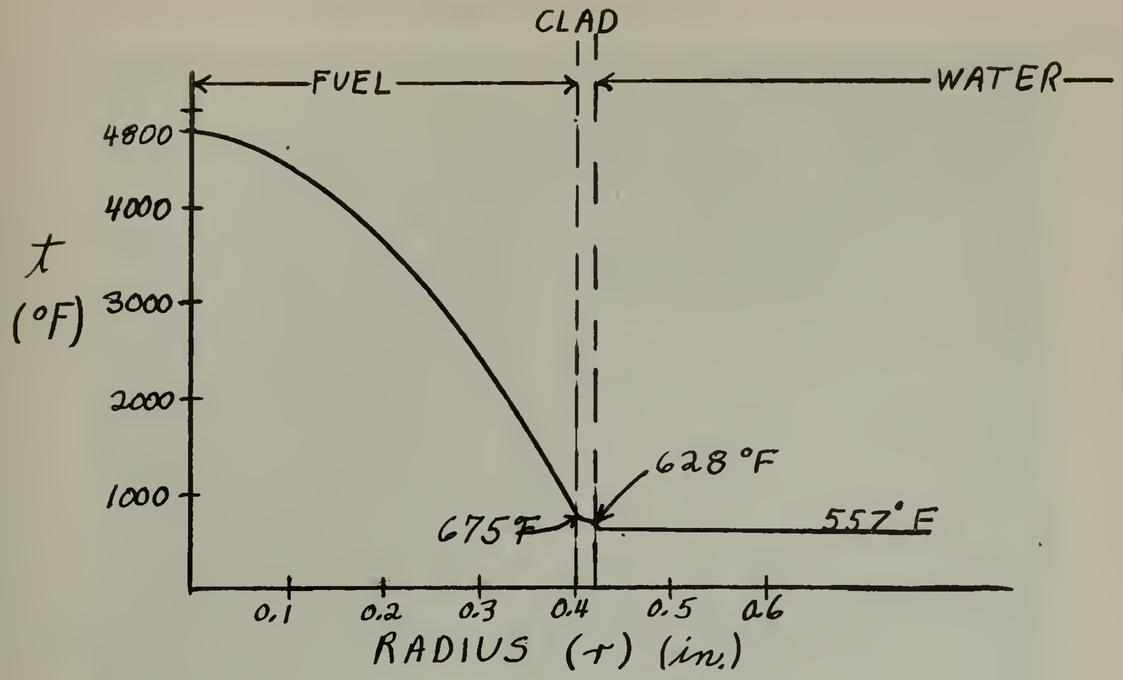


Figure 1

Assuming that the flux in the fuel can be expressed as:[1.]

$$\phi = AI_0(\kappa_0 r) \qquad (1)$$

then the heat generation pattern will be the same:

$$q(r) = AI_0(\kappa_0 r) \qquad (2)$$

where q(r) is the volume rate of heat generation at radius r. The average heat generation rate for the whole rod, $\bar{q}$, can be expressed as:

$$\bar{q}\pi R_0^{\,2} = \int_0^{R_0} 2\pi r\, AI_0(\kappa_0 r)\,dr$$

$$= \frac{2\pi A}{\kappa_0} \cdot R_0\, I_1(\kappa_0 R_0)$$

and

$$q(r) = \frac{\bar{q}\,\kappa_0 R_0}{2I_1(\kappa_0 R_0)}\; I_0(\kappa_0 r) \qquad (3)$$

The total heat generation Q(r), inside any radius, r, will be:

[1.] Glasstone and Edlund, 9.57

Figure 2
Total Mass of Uranium

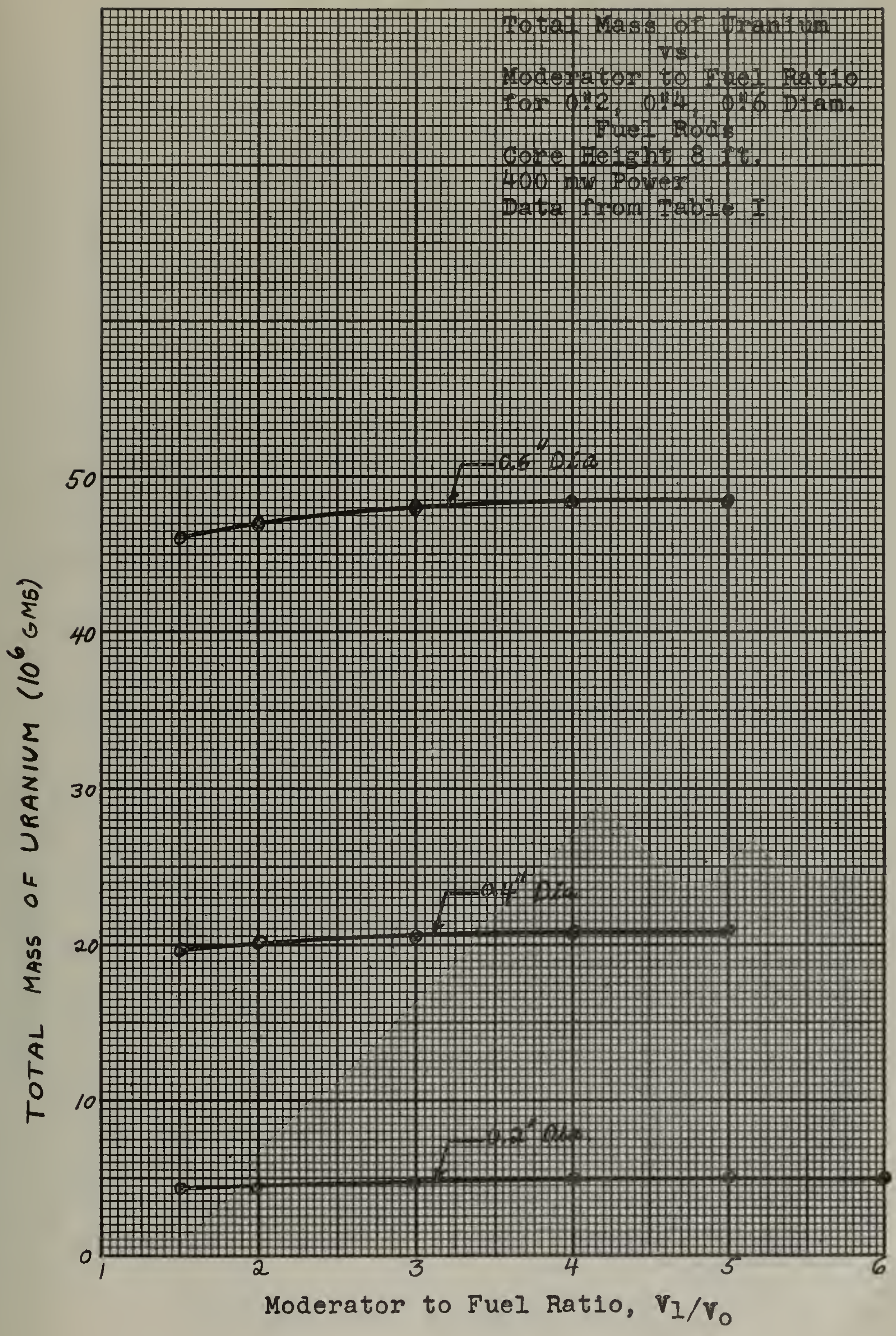


Figure 3

Enrichment vs Moderator to Fuel Ratio
for Stainless Steel Clad

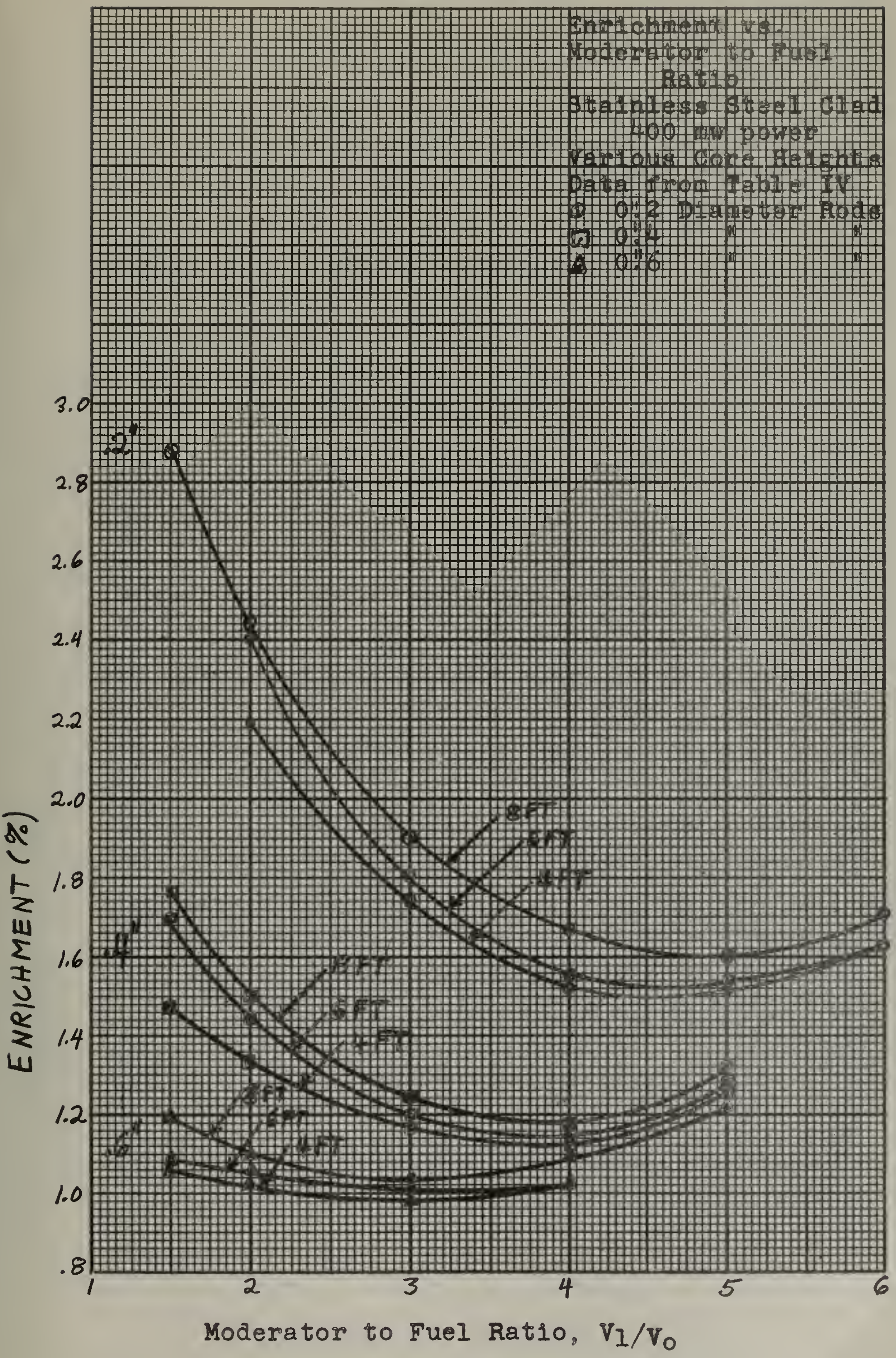
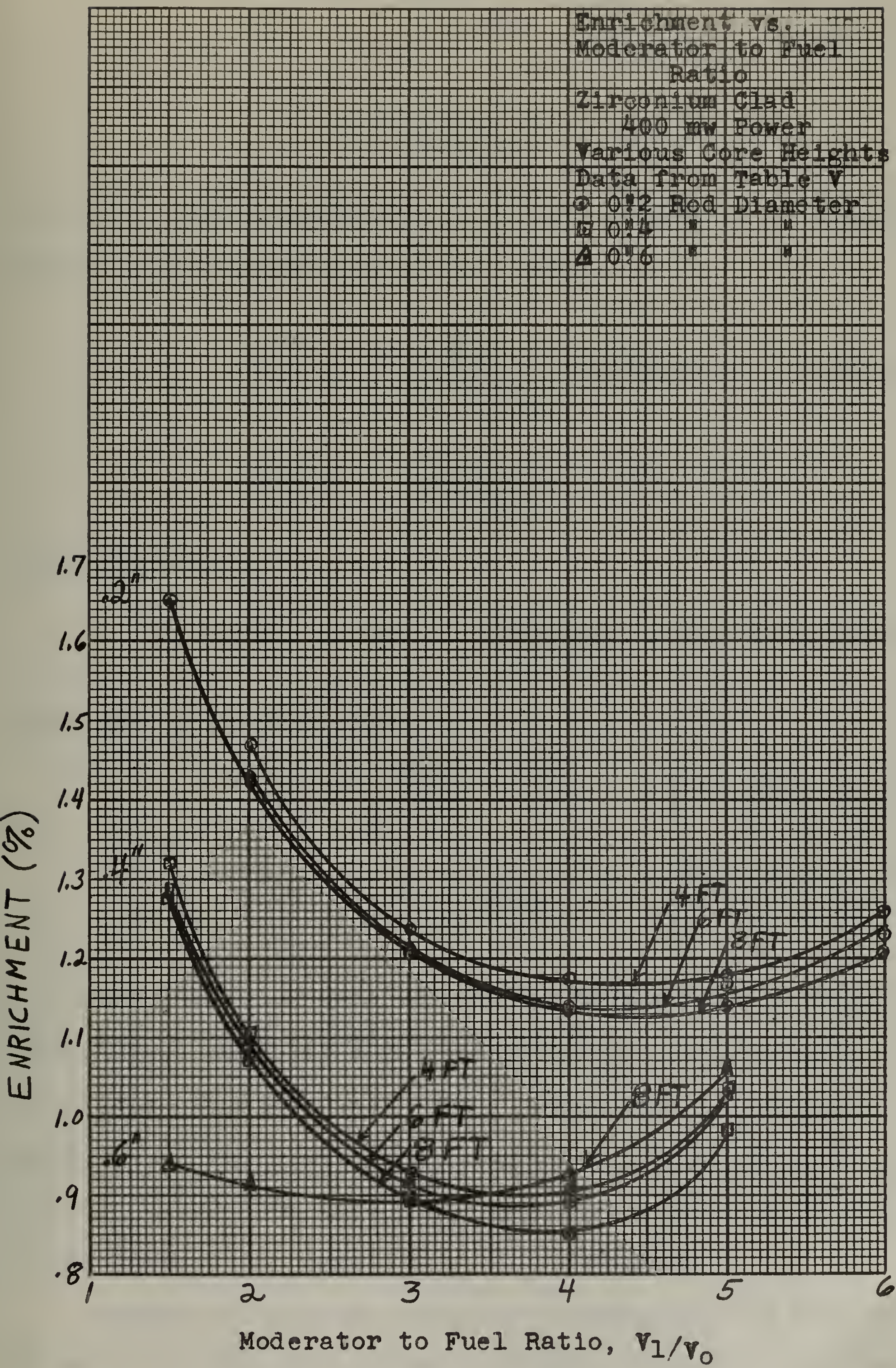


Figure 4

Enrichment vs Moderator to Fuel Ratio for Zirconium Clad



$$Q(\ ) = \int_0^r 2\pi r \frac{\bar{q}\,\kappa_0 R_0}{2I_1(\kappa_0 R_0)} I_0(\kappa_0 r)\,dr$$

$$= \pi \frac{\bar{q}\,R_0}{I_1(\kappa_0 R_0)} r I_1(\kappa_0 r) \qquad (4)$$

For steady state conditions, all this heat must be transferred across the circumference at r. Thus

$$Q(r) = \pi \frac{\bar{q}\,R_0}{I_1(\kappa_0 R_0)} rI_1(\kappa_0 r) = -k.2\pi r.dt/dr$$

$$dt/dr = \frac{q\,R_0}{2I_1(\kappa_0 R_0)} \cdot \frac{1}{k} \cdot I_1(\kappa_0 r) \qquad (5)$$

and integrating:

$$t_0 - t(R_0) = \Delta t = \frac{\bar{q}\,R_0}{2I_1(\kappa_0 R_0)} \times \frac{1}{kr} \times \left[I_0(\kappa_0 R_0) - 1\right] \qquad (6)$$

Since the total heat generated, Q, is $\bar{q}\,\pi R_0^2$,

$$\Delta t = \frac{QR_0}{\pi R_0^2} \times \frac{1}{2kr} \times \left[\frac{I_0(\kappa_0 R_0) - 1}{I_1(\kappa_0 R_0)}\right]$$

$$\Delta t = Q \times \frac{1}{2\pi k} \times \left[\frac{I_0(\kappa_0 R_0) - 1}{I_1(\kappa_0 R_0)}\right] \qquad (7)$$

The third factor in equation (7) is very nearly constant over the range of $\kappa_0 R_0$ involved, ranging from 0.499 at $\kappa_0 R_0 = 0.05$ to 0.459 at $\kappa_0 R_0 = 0.40$. Thus, since $2\pi k$ is constant, for constant Δt, Q is the same regardless of the value of R_0, and very nearly the same power per unit length is developed in the central rod regardless of its diameter.

Figure 5 shows the effect of increasing the radius of the core for the same central rod heat generation, assuming

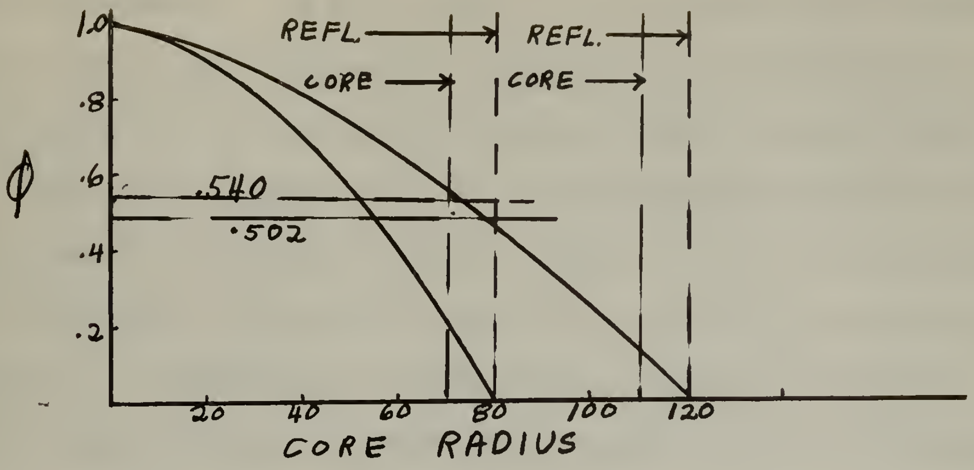


Figure 5

a J_0 overall flux distribution. The average to maximum heat generation ratio is reduced slightly, due to the large number of rods near the outer circumference which operate at low powers. This difference is, however, small; the ratios vary between about 0.5 and 0.6.

The two influences that will tend to increase the core radius for constant power output are:

1. Increasing the volume ratio of moderator to fuel.
2. Decreasing the core height.

The result is the pattern observed in Figure 2 and in Table I of the Results: since very nearly the same power per unit length can be developed in the average rod regardless of rod diameter, very nearly the same number of rods is required at a given core height regardless of rod diameter,

and the mass of uranium increases roughly as the square of the rod diameter. In the range of core parameters investigated, which represents about the range of practicality, the number of rods for given core height varied through about 20%. And since increase of the moderator-fuel ratio increases the diameter, a slightly larger number of rods is required, and, therefore, the mass of uranium is slightly increased. Similarly, a reduction in core height demands an increase in the number of the shorter rods, resulting in a diameter increase, and, again, a lowering of the average to maximum heat generation ratio and a slight increase in the mass of uranium.

The variation of enrichment with rod diameter and moderator to fuel ratio is a somewhat more complex phenomenon.

The effect of leakage is very small, for two reasons:

1. Both Fermi age, τ, and the squared diffusion length, L^2, are small for water moderated lattices. Room temperature values are 31 cm^2 and 4 cm^2 respectively; both increase with a decrease in water density, so that at the test conditions, the values are 49.5 cm^2 and 5 cm^2.
2. Due to the heat transfer requirements, the cores, of whatever proportion of length to diameter, are relatively large and the buckling consequently small. For the cores considered, B^2 computed from the core dimensions by[1].

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[1] Glasstone and Edlund, 7.60

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$$

varied between 3×10^{-4} and 9×10^{-4}.

From these values the non-leakage probability can be computed approximately from the modified one group theory discussed in the Introduction, as the product of slowing down and thermal diffusion non-leakages:

slowing down non-leakage $= e^{-B^2\tau}$

thermal diffusion non-leakage $= 1/(1 + L^2B^2)$

total non-leakage $= e^{-B^2\tau}/(1 + L^2B^2)$ (8)

For the range of bucklings and the appropriate age and diffusion length values, the non-leakage probability computed in this way for the cores considered varies between 0.95 and 0.985. This is so small a range as to be nearly constant. The machine calculation gives as a supplementary result the value of k_{oo} required for criticality; these are shown in Tables VII and VIII of the Results; it will be seen that they vary, for stainless steel cladding, from about 1.06 to about 1.13. Since

k (effective) $= 1 = k_{oo} \times$ non-leakage (9)

the non-leakage computed by this somewhat more sophisticated method varies in the narrow range from 0.88 to 0.94, still very nearly constant.

This means that leakage alone will not explain the strong variations found in enrichment, which must then depend on the effects of the cell parameters. These effects can be explained qualitatively with the use of a somewhat simplified

model, as described below.

As discussed in the Introduction, k_{oo} is the product of four factors; a fifth was introduced to bring the computer results into agreement with experiment,[1] so that

$$k_{oo} = 1.1\,\eta\varepsilon p f$$

Of these factors η is a constant for U-235 fuel

$$k_{oo} = 1.1 \times 2.08\,\varepsilon p f$$

Of the remaining three factors, ε and p were computed in the work preliminary to entering the machine calculation[2] and f was, in effect, computed by the machine. Thus,

$$f = \frac{1}{1.1 \times 2.08} \times \frac{k_{oo}}{\varepsilon p} = 0.436\,\frac{k_{oo}}{\varepsilon p} \qquad (10)$$

By equation (8),

$$k_{oo} = 1/(\text{total non-leakage})$$

And, using an average value of 0.90 for non-leakage,

$$f = 0.436 \times \frac{1}{0.90} \times \frac{1}{\varepsilon p} = \frac{0.485}{\varepsilon p}. \qquad (11)$$

The thermal utilization, f, is also definied as the ratio of absorption in the fuel to total absorption in fuel, U-238, and moderator:

$$f = \frac{V_o\,\Sigma_{a25}\,\bar{\phi}_o}{V_o\,\Sigma_{a25}\,\bar{\phi}_o + V_o\,\Sigma_{a28}\,\bar{\phi}_o + V_1\,\Sigma_{a1}\,\bar{\phi}_1} \qquad (12)$$

Algebraic manipulation of this gives

$$\frac{1}{f} = \frac{V_o\,\Sigma_{a25}\,\bar{\phi}_o}{V_o\,\Sigma_{a25}\,\bar{\phi}_o} + \frac{V_o\,\Sigma_{a28}\,\bar{\phi}_o}{V_o\,\Sigma_{a25}\,\bar{\phi}_o} + \frac{V_1\,\Sigma_{a1}\,\bar{\phi}_1}{V_o\,\Sigma_{a25}\,\bar{\phi}_o}$$

$$\frac{1}{f} = 1 + \frac{N_{28}}{N_{25}} \times \frac{\sigma_{28}}{\sigma_{25}} + \frac{V_1}{V_o} \times \frac{\bar{\phi}_1}{\bar{\phi}_o} \times \frac{\Sigma_{a1}}{\Sigma_{a25}} \qquad (13)$$

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1. See Appendix H.
2. See Procedure and Appendix D.

For low enrichments,

$$\text{enrichment} = N_{25}/(N_{25} + N_{28}) \simeq N_{25}/N_{28}$$

And for enrichments in a narrow range around 2% and cladding volume much smaller than water volume,

$$\Sigma_{a1}/\Sigma_{a25} \simeq \text{constant} \simeq 0.1$$

Substituting into equation (13)

$$\frac{1}{f} = \left[1 + 0.1 \times \frac{V_1}{V_o} \times \frac{\bar{\Phi}_1}{\bar{\Phi}_o}\right] + \frac{4.15 \times 10^{-3}}{\text{enrichment}} \qquad (14)$$

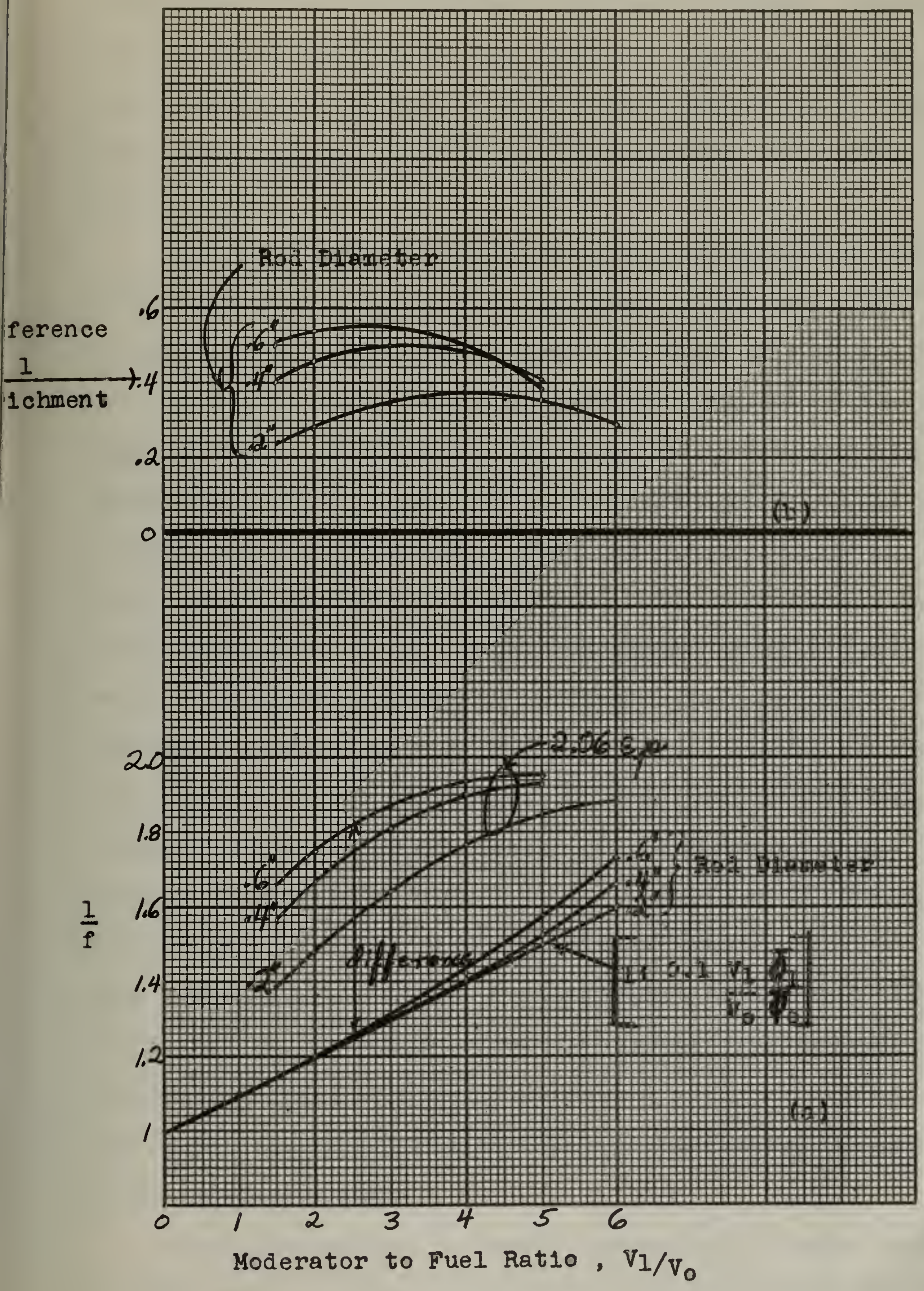
And from equation (11)

$$\frac{1}{f} = \frac{\varepsilon p}{0.485} = 2.06\,\varepsilon p \qquad (15)$$

Equation (15) may be plotted, as in Figure 6a from the computed data on fast fission factor and resonance escape probability, which are functions only of rod size and moderator to fuel ratio.

The two terms of equation (14) can be separated; the first term is a straight line of slope 0.1 with increasing volume ratio, slightly modified by the disadvantage factor, $\bar{\Phi}_1/\bar{\Phi}_o$, which was also computed in the preliminary calculations for each core and increases slowly with increasing rod diameter and volume ratio. This is also plotted in Figure 6a. The difference between the two lines so plotted for each rod size must be the second term of equation (14), and therefore proportional to the reciprocal of enrichment. It will be seen from Figure 6b, in which these differences are plotted for the rod diameters against the moderator to

Figure 6
Effect of Cell Parameters on
Enrichment (Equations (14) and (15))



fuel ratio, that the difference reaches a maximum, representing a minimum enrichment, for each rod diameter, and that these maxima are reached at a different moderator to fuel ratio for each rod diameter, which increases with decreasing rod diameter.

The same pattern can be seen in the results of the more accurate machine computation, as shown in Figures 3 and 4 and in Tables IV and V of the Results. Optimum enrichments are found for each rod diameter at moderator to fuel ratios which increase with a decrease in rod diameter. If leakage is assumed small and constant, the enrichment depends on the cell effects shown in Figure 6a. The product εp is greater for larger rod diameters, and increases with decreasing slope as the moderator to fuel ratio increases. However, since water is a strong absorber of neutrons, the absorption effect increases steadily as more water is added, tending to resist the advantage of increased εp. In addition, as rod size increases, the disadvantage factor effect becomes larger, also tending to resist the effect of εp. As a result, an optimum moderator to fuel ratio is reached, beyond which a further increase will result in an increased enrichment. Due to the earlier leveling off of εp and the greater effect of disadvantage factor, both of which can be clearly seen in Figure 6a, this optimum occurs at lower values of the volume ratio for larger rods.

That this effect is nearly independent of core geometry at this power is shown in Figures 7 and 8a in which enrichments

FIGURE 7
ENRICHMENT VS. CORE DIAMETER TO HEIGHT RATIO
STAINLESS STEEL CLAD

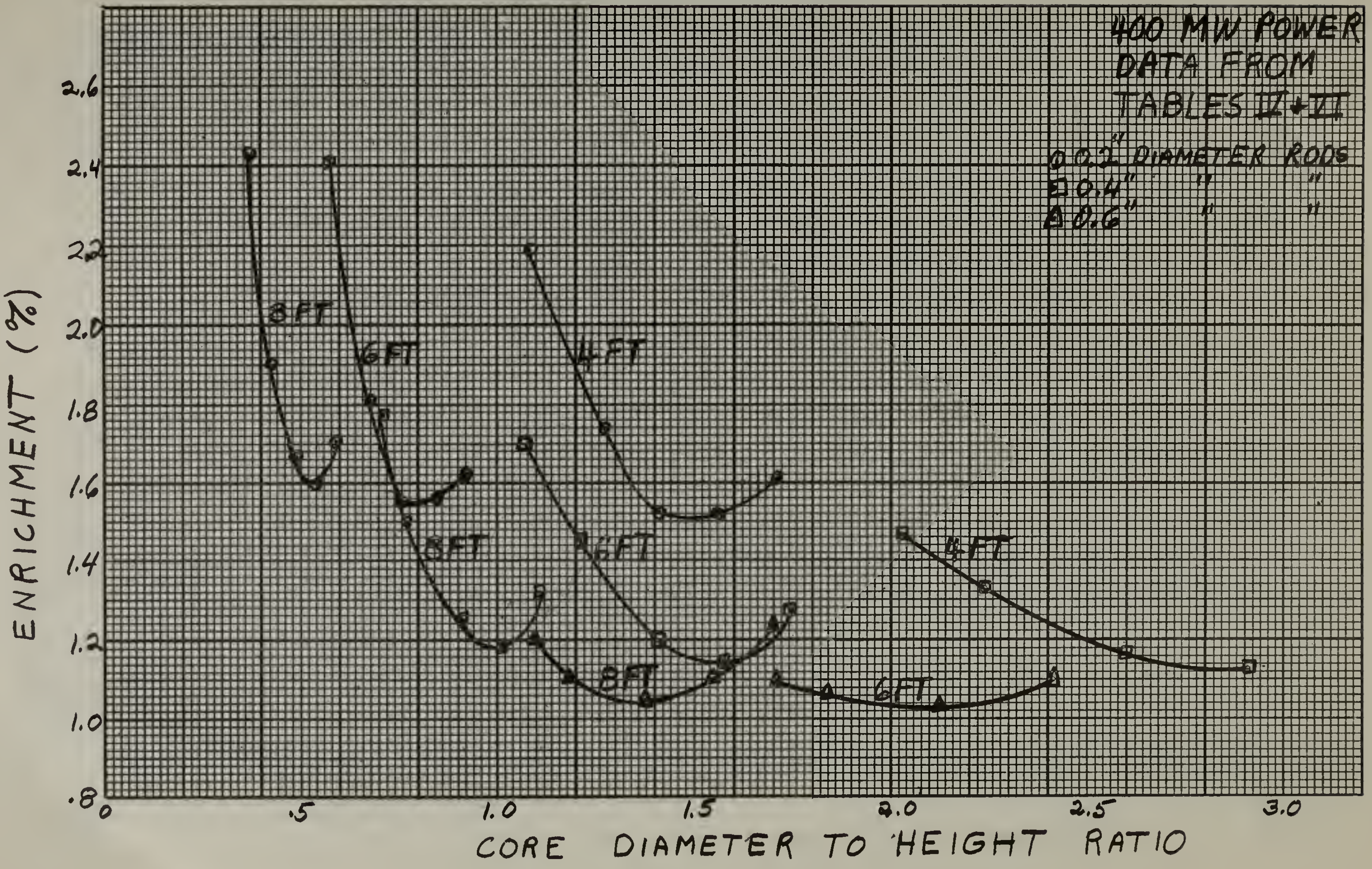
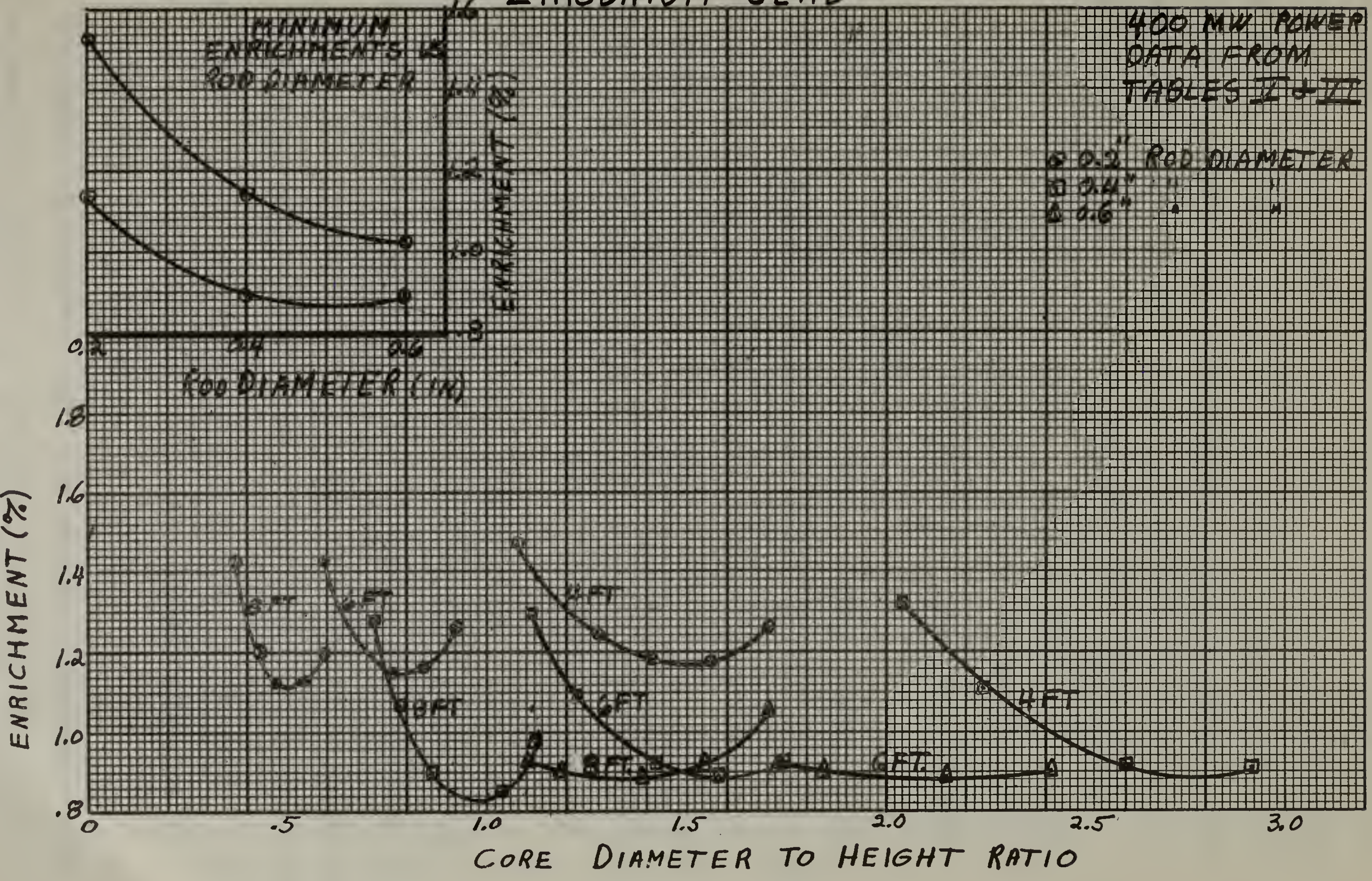


FIGURE 8

ENRICHMENT vs. CORE DIAMETER TO HEIGHT RATIO

ZIRCONIUM CLAD



are plotted for rod diameters and core heights against the diameter to height ratio of the core. If core geometry had a significant effect, it would be shown by the existence of a minimum enrichment near a diameter to length ratio of one. Although slight variations exist in the best enrichment obtainable with a given rod diameter, they are significant mainly in that they are so small. As indicated by Figures 7, 8a and 8b, an optimum enrichment can be obtained for a given rod diameter which is independent of the proportions of the core over a wide range.

Although the volume of cladding is small compared to those of fuel and water, its significance is seen in the comparison between the stainless steel and the zirconium clad fuels, shown in Figures 3, 4, 7 and 8. Due to the much smaller neutron absorption in zirconium ($\Sigma_a = 0.00526$ compared to $\Sigma_a = 0.1535$ for stainless steel), criticality can be attained with considerably lower enrichments. That the optimum volume ratios are not far removed from those found for the stainless steel clad cores is due to the lowering of Σ_{a25} with the decrease in enrichment from about 2% to about 1%. Thus Σ_{a1}/Σ_{a25}, the slope of the lower group of lines in Figure 6b before the disadvantage factor correction is applied, is not greatly changed, and the optimum enrichments occur at roughly the same values of V_1/V_0.

It should be observed that the minimum enrichment does not necessarily indicate the optimum core. The uranium cost

is a function both of enrichment and the total quantity of uranium. Since the total quantity of uranium goes up rapidly with the rod diameter and the enrichment is reduced only slowly, Atomic Energy Commission price figures released in December of 1956[1] indicate that the increased amount of uranium required for the larger rods will overweight the lower unit cost due to the lower enrichment, making the fuel cost less for the 0.2 inch rods than for the 0.6 inch. This effect will be reinforced by the greater amount of cladding material required to place an equal thickness of clad around the larger rods, since the number of rods is roughly the same. The cost of rod fabrication and of the pressure vessel are also significant items.

Finally, the enrichments computed are those required to reach initial criticality only. In order to compensate for the burnup effects of operation at power over a long period of time, these enrichments would have to be increased by a factor which depends on the lattice parameters. A method of finding this factor is derived in Appendix J. The cores computed in the thesis would be the basis of such a calculation.

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[1] "Enriched Uranium Prices", Nucleonics 14, 12, R-2 (Dec. 1956)

V.
CONCLUSIONS

I. Relating to methods of calculation.

1. Minimum core size can be computed for a specified power output and specified unit cell parameters.
2. The Whirlwind program written by J. R. Powell using a Fourier method of solution for critical mass of a cylindrical, highly enriched, homogeneous, heavy water moderated reactor can be adapted for use in the cylindrical, low enrichment, heterogeneous, light water case, and experimental correlations were obtained for the adjusted program.
3. The factor by which critical enrichment of a particular core must be increased to attain a specified power time of operation can be computed as a function of the critical amount of U-235 and the cell parameters.

II. Relating to the series of 400 megawatt thermal output cylindrical pressurized light water reactor cores studied.

1. Total mass of uranium required is a function of the heat transfer characteristics of the unit cell; it is approximately proportional to the square of the rod diameter.
2. Enrichment required for initial criticality is principally a function of the unit cell parameters and is nearly independent of the core size and shape, between diameter to length of core ratios of 0.8 and 3.
3. For each rod diameter and cladding material an optimum

moderator to fuel volume ratio exists at which the critical enrichment is a minimum. For the diameters tested, these were as follows:

Rod Diameter	Optimum V_1/V_0	
	Stainless Steel clad	Zirconium clad
0.6 inch	3.0	3.0
0.4 inch	3.75	4.0
0.2 inch	4.5	4.5

4. An optimum rod diameter at which minimum critical enrichment is lowest exists for rods clad with each cladding material. For zirconium clad rods, this occurs at about 0.45 inches rod diameter; for stainless steel clad, at slightly over 0.6 inches.

Rod Diameter	Minimum critical enrichment	
	Stainless Steel clad	Zirconium clad
0.6 inch	.99	.890
0.4 inch	1.12	.85
0.2 inch	1.5	1.13

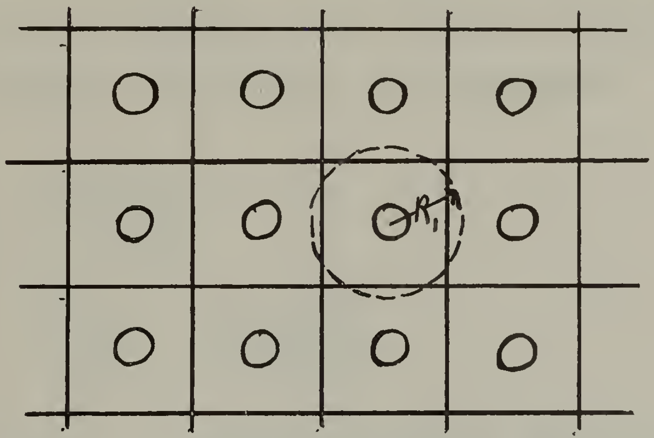
5. Critical enrichments are lower for zirconium clad rods than for stainless steel clad, of the order of 1% enrichment compared to the order of 2%.
6. Optimum critical enrichment does not necessarily indicate the optimum core; other factors which must be considered are total quantity of uranium, cost data for uranium, cladding materials, and fabrication of fuel elements and pressure vessels, and the additional enrichment necessary to attain a specified core life.

APPENDIX A

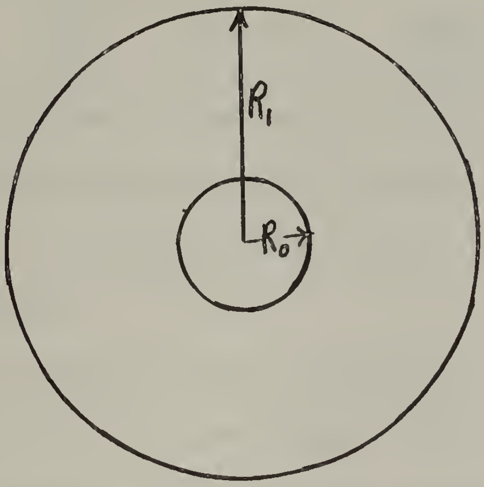
DISADVANTAGE FACTOR

A. L. Kaplan in a Massachusetts Institute of Technology S.M. thesis titled "Theoretical Studies of Neutron Flux Distribution Expected in the MIT Nuclear Research Reactor" compares the disadvantage factor calculated by two group diffusion theory based upon three reactor models. The first model assumes fast neutron flux constant through the cell and no net leakage of thermal neutrons across the outer boundary of the cell. This is identical to the development of Glasstone and Edlund. Model number two allows the fast neutron flux to vary throughout the cell as a function of the radial distance from the center of the fuel element and again assumes no net leakage of thermal neutrons across the outer cell boundary. The last model differs from the second by assuming a net leakage of thermal neutrons such that the ratio of thermal leakage to fast leakage equals the ratio for an equivalent homogenous cell. He concluded that the third model gave only three percent increase in accuracy over the first model and that this added accuracy did not justify the increased complexity in the calculation. This thesis work by Kaplan on heavy water, highly enriched cores and the close agreement between calculated and experimental disadvantage factors for light water, low enrichment cores (Table A-I) indicated that the one group diffusion theory in Glasstone and Edlund was adequate for this thesis.

This development approximates the reactor core by dividing it into a number of identical unit cells so that a square cross section is replaced by a circular cross section of equal area with radius R_1 in the manner shown in the following figure.



The equivalent cell then consists of a central fuel rod of radius R_0 with a concentric volume of moderator (water and clad) as shown below.



The Glasstone and Edlund development assumes slowing down density constant in the moderator and zero in the uranium. With the boundary conditions of continuity of neutron flux and neutron current density at the fuel rod-moderator boundary and no leakage at the outer cell boundary, the following equations for thermal utilization are developed:

$$1 + \frac{V_1 \Sigma_{a1}}{V_0 \Sigma_{a0}} F + (E - 1) \tag{1}$$

$$F = \frac{\chi_0 R_0}{2} \; \frac{I_0 (\chi_0 R_0)}{I_1 (\chi_0 R_0)} \tag{2}$$

$$E = \frac{\chi_1 (R_1^2 - R_0^2)}{2 R_0} \left[\frac{I_0(\chi_1 R_0) K_1(\chi_1 R_1) \quad K_0(\chi_1 R_0) I_1(\chi_1 R_1)}{I_1(\chi_1 R_1) K_1(\chi_1 R_1) - K_1(\chi_1 R_1) I_1(\chi_1 R_0)}\right] \tag{3}$$

By definition thermal utilization is expressed by

$$f = \frac{\Sigma_{a0} V_0 \bar{\Phi}_0}{\Sigma_{a0} V_0 \bar{\Phi}_0 + \Sigma_{a1} V_1 \bar{\Phi}_1} \tag{4}$$

Eliminating thermal utilization from equations (1) and (4)

$$\frac{\bar{\Phi}_1}{\bar{\Phi}_0} = F + (E-1) \; \frac{\Sigma_{a0} V_0}{\Sigma_{a1} V_1} \tag{5}$$

where $\bar{\Phi}_0$ and $\bar{\Phi}_1$ are the volume average thermal fluxes in the fuel and moderator, respectively, and their ratio is the disadvantage factor. This appendix contains a sample calculation for this factor.

In order to determine the accuracy of this method of calculation, a series of experimental flux traverses for light water moderated, low enrichment uranium lattices obtained at Brookhaven National Laboratory by H. J. Kouts, et. al. was utilized for comparison. Tables A-II, A-III and A-IV

and Figure A-I are excerpts from these reports. Table A-I compares calculated and experimental disadvantage factors over the range of experimental flux traverses presently available. It should be noted that this is an extremely narrow range; three enrichments, 1.027%, 1.15% and 1.30% for 0.6 inch rods and one enrichment, 1.027%, for 0.75 inch rods.

Agreement between calculated and measured disadvantage factors for the 0.6 inch rods are all less than 6% error. With increased diameter the percent error becomes greater. This is explained by the failure of the theoretical functions describing flux within the rod, the bessel function $I_0(\kappa_0 r)$, and flux within the moderator $AI_0(\kappa_1 r) + BK_0(\kappa_1 r)$ to match the experimental flux traverses for the 0.75 inch rods as shown by Figure A-3. Brookhaven National Laboratory found it necessary to multiply the κ of the fuel rod by an average value of approximately 1.5 and the κ of the moderator by 1.25 to match theoretical with experimental flux traverses. When these adjustments to κ are made, the calculated value is within four percent of the measured value of the disadvantage factor for 0.75 inch rods.

From this limited comparison it appeared that the one group diffusion theory with zero neutron leakage from the unit cell would give sufficiently accurate answers at very low enrichments and at diameters below 0.6 inch. However, as enrichment and rod diameter increase, the disadvantage factors become increasingly less accurate. For the 0.75 inch case the previously mentioned arbitrary factors could be applied;

but until more experimental work is done with enrichments above 3%, larger, undetermined errors will probably exist in the disadvantage factors computed by this method.

Sample Calculation for Disadvantage Factor

Stainless Steel Clad, UO_2 Fuel, H_2O Moderator of .79 gm/cm^3 density

$V_1/V_0 = \underline{2.0}$ $\quad R_0 = \underline{.508}$ cm

$R_1 = R_0\sqrt{V_1/V_0+1}$ $\quad \underline{.880}$

$V_0 = \pi R_0^2$ $\quad \underline{.810}$

$V_c = (.239)(R_0+.019)$ $\quad \underline{.126}$

$V_{H_2O} = \pi R_1^2 - (V_0+V_c)$ $\quad \underline{1.494}$

$V_1 = V_{H_2O} + V_c$ $\quad \underline{1.62}$

1.66 V_{H_2O}/V_1 $\underline{1.53}$

.89 V_c/V_1 $\underline{.07}$

Σ_{tr1} 1.60

.212 (1-enrich.) $\underline{.2065}$

2.16 (enrich.) $\underline{.0432}$

Σ_{tr0} .2497

.0115 V_{H_2O}/V_1 $\underline{.0106}$

.1535 V_0/V_1 $\underline{.0119}$

Σ_{a1} .0225

.0404(1-enrich.) $\underline{.0395}$

9.65 (enrich.) $\underline{.1931}$

Σ_{tr1} .2326

$\kappa_1 = \sqrt{3\Sigma_{tr1}\Sigma_{a1}} = \underline{.33}$

$\kappa_0 = \sqrt{3\Sigma_{tr0}\Sigma_{a0}} = \underline{.415}$

$x = \kappa_1 R_0 = .1675$

$\kappa_0 R_0 = .211$

$1/ = \kappa_1 R_1 = .290$

Fig. 3 $(E-1) = .006$

Fig. 1 $F = 1.0054$

$$\frac{\bar{\Phi}_1}{\bar{\Phi}_0} = F + (E-1)\frac{\Sigma_{a0} V_0}{\Sigma_{a1} V_1} = 1.036$$

Figure A-I

Intracell Flux Distributions of Water-Uranium Lattices

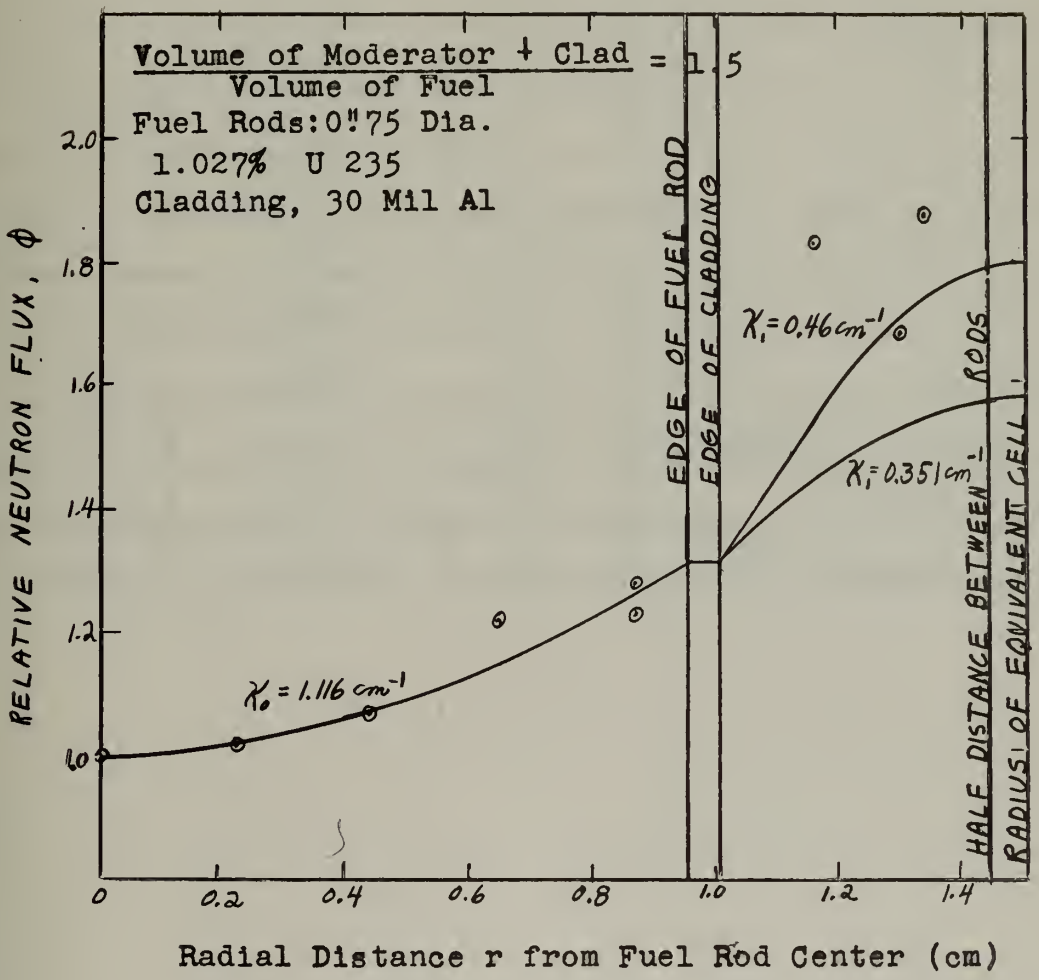


TABLE A-I

COMPARISON OF CALCULATED AND MEASURED DISADVANTAGE FACTORS

Rod Diam.	Enrichment	V_W/V_U[1]	Calc. DA	Meas. DA	% Error
0.6	1.30 %	3	1.32	1.40	5.7
0.6	1.30 %	2	1.27	1.30	2.3
0.6	1.30 %	1.5	1.22	1.295	5.8
0.6	1.15 %	3	1.32	1.36	3.0
0.6	1.15 %	2	1.26	1.33	5.3
0.6	1.15 %	1.5	1.21	1.27	4.7
0.6	1.027%	3	1.29	1.32	2.3
0.75	1.027%	2	1.33	1.595	16.6
0.75	1.027%	2	1.565[2]	1.595	1.8
0.75	1.027%	4	1.5	1.73	13.3
0.75	1.027%	4	1.80[2]	1.73	4.0

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[1] Kouts defines V_W/V_U as the ratio of water to uranium for 0.6 inch rods and as the ratio of water plus clad to uranium for 0.75 inch rods.

[2] Using adjusted kappa values of 1.5 κ_0 for fuel and 1.25 κ_1 for moderator.

TABLE A-II

0.6 Inch Diameter, 1.15% Enrichment, Aluminum Clad Uranium Rods in Light Water (BNL 1987)

V_w/V_u	$\bar{\Phi}_w$	$\bar{\Phi}_u$	$\bar{\Phi}_{al}$	f [1.]
1.5	1.449	1.127	1.280	.917
2	1.527	1.109	1.250	.888
2	1.514	1.124	1.252	.890
3	1.565	1.135	1.290	.842
4	1.643	1.129	1.285	.793

$\Sigma_u = .445$

$\Sigma_w = .0195$

$\Sigma_{al} = .0115$

[1.] Dividing the moderator into cladding and coolant, thermal utilization is defined as

$$f = \frac{\bar{\Phi}_u \Sigma_u V_u}{\bar{\Phi}_u \Sigma_u V_u + \bar{\Phi}_w \Sigma_w V_w + \bar{\Phi}_{al} \Sigma_{al} V_{al}}$$

TABLE A-III

0.6 Inch Diameter, 1.027% Enrichment, Aluminum Clad Uranium Rods in Light Water (BNL 1796)

V_w/V_u	$\bar{\Phi}_u$	$\bar{\Phi}_w$	$\bar{\Phi}_{al}$	f[1]
1	1.079	1.271	1.172	.941
1.5	1.105	1.381	1.211	.913
2	1.121	1.443	1.246	.886
3	1.107	1.505	1.211	.833
4	1.121	1.610	1.252	.781

[1] $\Sigma_u = .4096$

$\Sigma_w = .0195$

$\Sigma_{al} = .0115$

TABLE A-IV

0.6 Inch Diameter, 1.3% Enriched, Aluminum Clad Uranium Rods in Light Water (BNL 1783) Values from graph.

V_w/V_u	$\bar{\Phi}_w$	$\bar{\Phi}_u$	$\bar{\Phi}_{al}$
1	1.35	1.125	1.25
1.5	1.45	1.15	1.32
2	1.55	1.175	1.35
3	1.75	1.2	1.40

APPENDIX B

METHOD OF ADJUSTING HOMOGENEOUS PROGRAM CONSTANTS TO OBTAIN CRITICAL MASS OF A HETEROGENEOUS REACTOR

The disadvantage factor was utilized to adjust homogeneous program constants to give a critical mass of a heterogeneous reactor. Each value for a thermal cross section is adjusted to have the same value as the effective volume average value of the corresponding cross section in the heterogeneous reactor weighted by the relative flux throughout the unit cell in the following manner.

$$\bar{\Sigma}_a(25) = \frac{N(25)\sigma_a(25)\, V_0 \bar{\Phi}_0}{V_0 \Phi_0 + V_1 \Phi_1} = N'(25)\sigma_a'(25)$$

where N(25) are the atoms per cubic centimeter of fuel and N'(25) are the atoms per cubic centimeter of unit cell volume.

$$N'(25) = \frac{N(25)\, V_0}{V_0 + V_1}$$

Therefore

$$\sigma_a'(25) = \left(\frac{V_0 + V_1}{V_0 + V_1 \dfrac{\bar{\Phi}_1}{\bar{\Phi}_0}} \right) \sigma_a(25)$$

This factor was used to adjust both U-235 and U-238 absorption and transport cross sections.

Light water cross sections were adjusted in an analogous manner

$$\bar{\Sigma}_a(w) = \frac{N(w)\sigma_a(w)\, V_1 \bar{\Phi}_1}{V_0 \Phi_0 + V_1 \Phi_1} = N'(w)\sigma_a'(w)$$

$$N'(w) = \frac{N(w)\, V_1}{V_0 + V_1}$$

$$\sigma_a'(w) = \left(\frac{V_0 + V_1}{V_0 \dfrac{\bar{\Phi}_0}{\bar{\Phi}_1} + V_1} \right) \sigma_a(w)$$

This factor was used to adjust water absorption and transport

cross sections.

Cladding cross sections were not adjusted because the flux in the clad is very nearly equal to the average flux in the unit cell. This is apparent by inspection of Figure A-I. This fact was checked by calculation by comparing values of $\bar{\Phi}_1/\bar{\Phi}_{AVG}$ with experimental values of $\bar{\Phi}_1/\bar{\Phi}_{al}$ where

$$\frac{\bar{\Phi}_1}{\bar{\Phi}_{AVG}} = \frac{V_0 + V_1}{V_0 \dfrac{\bar{\Phi}_0}{\bar{\Phi}_1} + V_1}$$

The error in all cases indicates that by not adjusting the cross sections for cladding a greater cladding absorption was being assumed. This was a result of the fact that the average flux in a unit cell was higher than the actual neutron flux in the clad. However, the error introduced is necessarily small because of the small percentage of volume that the clad occupies in a unit cell.

APPENDIX C

ADJUSTMENTS TO NUCLEAR CONSTANTS FOR POWELL FOURIER PROGRAM

The Powell program was written for a highly enriched, homogeneous, heavy water moderated reactor. The adjustment of the cross sections to obtain a heterogeneous critical mass is discussed in Appendix B. Table E-I lists the inputs on the data tape for Powell's cylindrical Fourier program.

In this thesis it was planned to utilize the program for low enrichment, light water power reactors. However, there was no provision for the handling of uranium-238 in the program. The best method appeared to be to continue to define η as $\nu \dfrac{\Sigma_f (25)}{\Sigma_a (25)}$ and to treat the uranium-238 as an additional cladding. By this method the absorption in the uranium-238 would appear in the four factor formula in the thermal utilization.

$$k_\infty = \varepsilon p \eta f$$

$$f = \frac{\text{thermal absorption in U-235}}{\text{total thermal absorption in core}}$$

The fast fission factor, ε, and the resonance escape probability, p, were combined with the η for uranium-235. This product $\varepsilon p \eta$ with an additional arbitrary factor of 1.1 to bring experimental and program critical masses into agreement, constituted the input for "η_{25}" in the Powell program.

The program, as written, included density and atomic weight for aluminum and heavy water as the cladding and moderator, respectively. In order to use the program for other materials,

it was necessary to either modify inputs to account for these constants or to modify the program by changing the inputs of the four drum addresses concerned. Both methods were utilized.

Method A combined the effect of uranium-238 and clad material in the following manner

$$"\sigma_a(\mathrm{al})" = \frac{\Sigma_{28}\left(\dfrac{V_0+V_1}{V_0+V_1\frac{\Phi_1}{\Phi_0}}\right)V_0 + \Sigma_{ac}V_c}{(V_0+V_c)(6.025\times10^{23})} \tag{3}$$

$$"\sigma_a(\mathrm{al})" = \frac{"\Sigma_a(\mathrm{al})"\,A}{6.025\times10^{23}} = \frac{"\Sigma_a(\mathrm{al})"}{6.025\times10^{23}} \tag{4}$$

$$"\sigma_a(D_2O)" = \left(\frac{V_0+V_1}{V_0\frac{\Phi_0}{\Phi_1}+V_1}\right)\sigma_a(H_2O) \tag{5}$$

The drum addresses were modified to read

DA 420/ 420/ $\rho_{D_2O} = \rho_{H_2O}$

DA 424/ 424/ $A_{D_2O} = +\ 18.$

DA 418/ 418/ $\rho_{al} = +\ 1.$

DA 422/ 422/ $A_{al} = +\ 1.$

This method used the macroscopic cross sections divided by Avogadro's number and modified the density and the atomic number of the clad to equal one so that when the computed N(al) and "σ(al)" were multiplied together an equivalent clad macroscopic cross section was obtained. Transport cross sections were computed identically.

Method B is a mathematically equivalent method and was devised to circumvent the modifying of drum addresses. This was done in the following manner:

$$\text{"}\sigma_a(al)\text{"} = \frac{N_c V_c \sigma_{ac} + N_{28} V_0 \sigma_a(28)\left(\frac{V_0+V_1}{V_0+V_1 \phi_1/\phi_0}\right)}{N_{al}(V_c+V_0)} \quad (6)$$

$$\text{"}\sigma_a(D_2O)\text{"} = \frac{\left(\frac{V_0+V_1}{V_0 \phi_0/\phi_1 + V_1}\right)\sigma_a(H_2O)\, N_{H_2O}}{N_{D_2O}} \quad (7)$$

$$N_{D_2O} = \frac{6.025 \times 10^{23}}{A}\rho = \frac{(6.025 \times 10^{23})(1.105)}{20} \quad (8)$$

Method B is merely a variation of Method A allowing the use of the program without modification to existing constants incorporated in the writing of the program. Method A, when used in the program, gave inconsistent low critical masses which led the authors to believe that the drum addresses were not being properly modified. This was confirmed by the subsidiary outputs of the program. The following page is a sample calculation for UO_2 fuel and light water moderator of density 0.79 gm/cm^3 employing method B.

The quantity "x" in the program was changed from the ratio of the volumes of aluminum to heavy water to the ratio of the sum of the uranium-238 and cladding material to light water. "x" is used in the program to homogenize the macroscopic cross section of cladding and moderator.

$$\bar{\Sigma} = \Sigma(w)\frac{1}{1+x} + \text{"}\Sigma(al)\text{"}\frac{x}{1+x} \quad (9)$$

$$\bar{\Sigma} = \Sigma(w)\left(\frac{V_w}{V_w+V_{28}+V_c}\right) + \text{"}\Sigma(al)\text{"}\left(\frac{V_{28}+V_c}{V_w+V_{28}+V_c}\right) \quad (10)$$

Sample Calculations for

Nuclear Data, UO_2 Fuel, H_2O Moderator, $\rho_w = .79$ (Method B)

	Σ_{1a}	Σ_{1tr}
28	.0404	.212
55	.1535	.890

$V_o = .810$ $\qquad$ $V_o = .810$

$V_c = \underline{.126}$ $\qquad$ $V_1 = \underline{1.623}$

$V_o + V_c = .936$ $\qquad$ $V_o + V_1 = 2.433$

$V_o + V_c \times .06025 = \underline{.0564}$ (A)

$\frac{\bar{\Phi}_1}{\bar{\Phi}_0} = 1.036$ $\qquad$ $V_1 \times \frac{\bar{\Phi}_1}{\bar{\Phi}_0} = 1.68$ $\qquad$ $V_o \frac{\bar{\Phi}_0}{\bar{\Phi}_1} = .783$

$$\frac{V_o + V_1}{V_o + V_1 \frac{\bar{\Phi}_1}{\bar{\Phi}_0}} = \frac{2.433}{2.49} = .976 \quad (B)$$

$$\frac{V_o + V_1}{V_o \frac{\bar{\Phi}_0}{\bar{\Phi}_1} + V_1} = \frac{2.433}{2.406} = 1.01 \quad (C)$$

$.0404 \times V_o \times (B) = .0319$

$.1535 \times V_c = \underline{.0194}$

$\underline{.0513}$ $\qquad = \underline{.910}$ $\qquad$ "σ_a Al"

$(A) = .0564$

$.212 \times V_o \times (B) = .168$

$.890 \times V_c = \underline{.112}$

$\underline{.280}$ $\qquad = \underline{4.97}$ $\qquad$ "σ_{tr} Al"

$(A) = .0564$

$.344 \times (C) = .347 = $ "σ_a D_2O"

$50.0 \times (C) = 50.5 = $ "σ_{tr} D_2O"

$$x = \frac{(.98)(.81) + .126}{1.494} = \frac{.921}{1.494} = .616$$

APPENDIX D

COMPUTATION FOR FAST FISSION FACTOR AND RESONANCE ESCAPE PROBABILITY

In order to account for the fissions in uranium-235 and uranium-238 produced by fast neutrons before slowing down to thermal energies, a fast fission factor is utilized. It is defined as the ratio of the total number of fast neutrons produced by fissions due to neutrons of all energies to the number resulting from thermal neutron fissions. Computed fast fission factors were calculated by using Hellen's empirical formulation:

$$\epsilon = 1 + \frac{.1565}{1 + 0.875\,\rho_w \frac{V_w}{V_u} + .288\,\frac{V_s}{V_u}} \qquad (1)$$

ρ_w = density of water

V_w = volume of water

V_u = volume of uranium

V_s = volume of structural materials

This factor was checked for eight critical experimental cases performed at the Bettis plant of the Westinghouse Corp. for which a summary is given in Table D-I. Errors of computed values are in all cases less than 1 percent of measured values.

That fraction of source neutrons which escape capture while being slowed down to thermal energy is represented by the resonance escape probability, p. The method outlined in Glasstone and Edlund for heterogeneous reactors was used with the empirical quantities from The Reactor Handbook, Volume I,

to compute resonance escape probability.

This method assumes that neutrons within the energy range where resonance absorption by fuel is important can be represented as a single energy group, using effective slowing down and absorption cross sections.

A resonance utilization analogous to the thermal utilization is defined as:

$$f_r = \frac{\text{rate of absorption of resonance neutrons}}{\text{rate of production of resonance neutrons}} \qquad (2)$$

$$f_r = \frac{V_0 (\bar{\phi}_0)_r \Sigma_{a0}}{V_0 (\bar{\phi}_0)_r \Sigma_{a0} + V_1 (\bar{\phi}_1)_r \Sigma_1} \qquad (3)$$

Equation (3) assumes that the leakage in the resonance group is small compared to absorption and slowing down. Resonance utilization may be expressed in terms of lattice parameters as:

$$\frac{1}{f_r} = 1 + \frac{V_1 \Sigma_1}{V_0 \Sigma_{a0}} F + (E-1) \qquad (4)$$

where F and E for cylindrical geometry are defined in Appendix A.

Resonance escape probability is written in terms of f_r as:

$$p = \exp\left(- \frac{f_r}{1 - f_r}\right) \qquad (5)$$

It has been assumed that the resonance neutrons can be approximated by one energy group. They exist over an energy range greater than would be predicted by one group diffusion theory because of the selective absorption of neutrons as they

pass through uranium-238 resonances enroute to the interior of the fuel rod.

To determine this effect average slowing down and absorption cross sections are computed based upon empirical relations derived from experiment. The microscopic average resonance cross section is expressed as:

$$\bar{\sigma}_{ao} = \frac{A + \mu \frac{S}{M}}{\ln E_2/E_1}$$

M = mass of fuel lump, grams

S = surface area of lump, cm^2

A = empirical constant, barns

μ = empirical constant, gm/cm^2

E_1 = lower energy limit of resonance

E_2 = upper energy limit of resonance

Table D-II lists empirical constants from The Reactor Handbook, Volume I, for natural uranium.

For cylindrical rods:

$$\frac{S}{M} = \frac{2\pi R_o h}{\rho \pi R_o^2 h} = \frac{2}{\rho R_o}$$

$$\bar{\Sigma}_{ao} = \frac{NA\left[1 + \frac{\mu}{A}\frac{S}{M}\right]}{\ln E_2/E_1} = \frac{NA\left[1 + \frac{\mu \, 2}{\rho A R_o}\right]}{\ln E_2/E_1}$$

The effective resonance slowing down cross section for the moderator may be expressed by:

$$\bar{\Sigma}_1 = \frac{\xi \bar{\Sigma}_s}{\ln E_2/E_1}$$

Table D-III lists the resonance constants for moderator from

The Reactor Handbook, Volume I.

A sample calculation sheet for the calculation of resonance escape probability for UO_2 fuel is shown on the following page.

Eight experimental cases were calculated and comparison of measured and computed values are shown in Table D-I. The maximum percent error from the measured p is 2.4%.

Sample Calculation for
Resonance Escape Probability UO_2 Fuel, H_2O Moderator (.79 gm/cm^3)
Core No. 39

Fuel Rod Radius, R_o = .508 cm

Unit Cell Radius, R_1 = .880 cm

$$\frac{.382}{R_o} = .753$$

$$\frac{+1.0}{1.753} \times .0353 = \underline{.0620} \quad \Sigma_{ao}$$

$$.42 \times R_o = .213 \qquad (\kappa_0 R_o)$$

$$.649 \times R_o = .329 \qquad (\kappa_1 R_o)$$

$$.649 \times R_1 = .570 \qquad (\kappa_1 R_1)$$

From Appendix A, F = 1.005

E = 1.026

$$\frac{V_w}{V_o} \times \frac{.140}{\Sigma_{ao}} \times F = \frac{1.494 \times .140 \times 1.005}{.810 \times .0620} = \underline{4.22} \quad (A)$$

$$f_r = \left[(A) + E\right]^{-1} = \left[4.22 + 1.026\right]^{-1} = .190$$

$$\frac{f_r}{1-f_r} = \frac{.190}{.810} = \underline{.235} \quad (B)$$

$$P = e^{-(B)} = .790$$

Fast Fission Factor

$$\varepsilon = 1 + \frac{.1565}{1 + .875\,\rho_w \frac{V_w}{V_\mu} \quad .288 \frac{V_s}{V_\mu}}$$

$$= 1 + \frac{.1565}{1 + (.875)(.79)\frac{1.494}{(.475)(.810)} + \frac{(.288)(.126)}{(.475)(.810)}} = 1.041$$

1.1)ηp = (1.1)(1.041)(.790)(2.08) = 1.885

TABLE D-I

COMPARISON OF MEASURED AND CALCULATED FAST FISSION FACTORS AND RESONANCE ESCAPE PROBABILITIES

Fuel	Rod Diam. in.	Clad	Enrich. %	V_w/V_u	Calc. ε	Meas. ε	%error ε	Calc. p	Meas. p	%error p
U	.6	Al	1.3	3/1	1.043	1.047	.4	.859	.850	1.1
U	.6	Al	1.3	2/1	1.056	1.061	.5	.796	.794	.3
U	.6	Al	1.3	1.5/1	1.066	1.072	.5	.737	.720	2.3
U	.6	Al	1.15	3/1	1.043	1.050	.7	.859	.839	2.4
U	.6	Al	1.15	2/1	1.056	1.065	.9	.796	.792	.5
UO_2	.6	Al	1.3	3.05/1	1.041	1.039	.2	.816	.810	.7
UO_2	.6	Al	1.3	4/1	1.033	1.033	0	.854	.835	2.2
UO_2	.6	Al	1.3	5/1	1.031	1.029	.2	.879	.870	1.0

TABLE D-II

EMPIRICAL RESONANCE CONSTANTS FOR FUEL[1]

Material	A(barns)	μ (gm/cm^2)	ln E_2/E_1	κ_o(cm-1)
U metal	9.25	8/3	5.6	0.42
UO_2	11.51	1.92	7.3	0.42

---

[1] Reactor Handbook, Vol. I

TABLE D-III

EMPIRICAL RESONANCE CONSTANTS FOR MODERATOR[1]

Material	Density (gm/cm^2)	$\xi\sigma_s$ barns/atom	$\bar{\kappa}_2/\rho$ cm^2/gm [2]
H_2O	1.0	38.5	.583

---

[1] Reactor Handbook, Vol. I.

[2] $\frac{\kappa_2}{\rho}$ is multiplied by .88 for UO_2 fuel.

APPENDIX E

POWELL CYLINDRICAL FOURIER PROGRAM INPUTS AND OUTPUTS

Figure E-I defines the dimensions for the six radial and five longitudinal regions for Powell's program. In this thesis two radial regions; a central core and radial reflector and three longitudinal regions; a central core and a top and bottom reflector were used. Thus R_1 and R_2 were effectively set equal to zero, i.e. 1 x 10^{-10}cm. R_3 was the core radius computed from heat transfer considerations. The thickness of the two outer reflectors was set equal to zero by letting $R_3 = R_4 = R_5$. R_6, the outer radius of the reactor, was then simply the sum of the core radius and reflector thickness. In every case it was assumed that a reflector thickness of eighteen centimeters was equivalent to an infinite reflector. The radial reflector thickness was set equal to eighteen centimeters. The thickness of the top and bottom reflectors (number two) was set equal to zero. The top and bottom reflector (number one) thickness was set equal to eighteen centimeters.

For the radial region chosen as the core, in this case region three; Fermi age, thermal diffusion coefficient, infinite multiplication factor, and thermal absorption cross sections are computed. Thus, the inputs to these regions are normally chosen as zero except for the thermal diffusion coefficient which appears in the denominator and must not have a zero value. This same situation exists for these quantities in the longitudinal properties. The core properties are

replaced by computed numbers as the program runs so that any number used to describe the core region will be erased.

The inputs in the nuclear constant section are discussed in Appendix C.

The final inputs are two k_{oo} guesses which are used to compute values of the critical equation. In general, neither of these guesses will satisfy the critical conditions, so the two assumed values of k_{oo} corresponding to the two computed values of the critical determinant are extrapolated linearly to the value of k_{oo} at which the value of the determinant is zero. This process is continued always throwing away the guess that is most in error until the difference between $M(25)_{n=3}$ and $M(25)_{n=2}$ is less than one tenth of a gram. The guesses need not be close to the answer except that the closer they are, the less time will be required for convergence.

Of particular interest when making a parametric study such as this is the fact that once the main program is fed into the computer, a series of data tapes may be run without reading in the main program again. The data tapes need not be complete, but need only be cut from the beginning through the numbers which were changed from the preceding data tape. All constants read in on the first parameter remain in storage until replaced by a new number, and all computed values in storage are erased prior to starting a new run. Thus when systematically varying one constant a great amount of time can be saved.

Table E-I is a sample data tape preparation form from

the J. Powell thesis using data from run 39. Table E-II is a sample of the form found convenient for typing this data on the Whirlwind data tape.

Figure E-I

Powell Fourier Program
Core Configuration

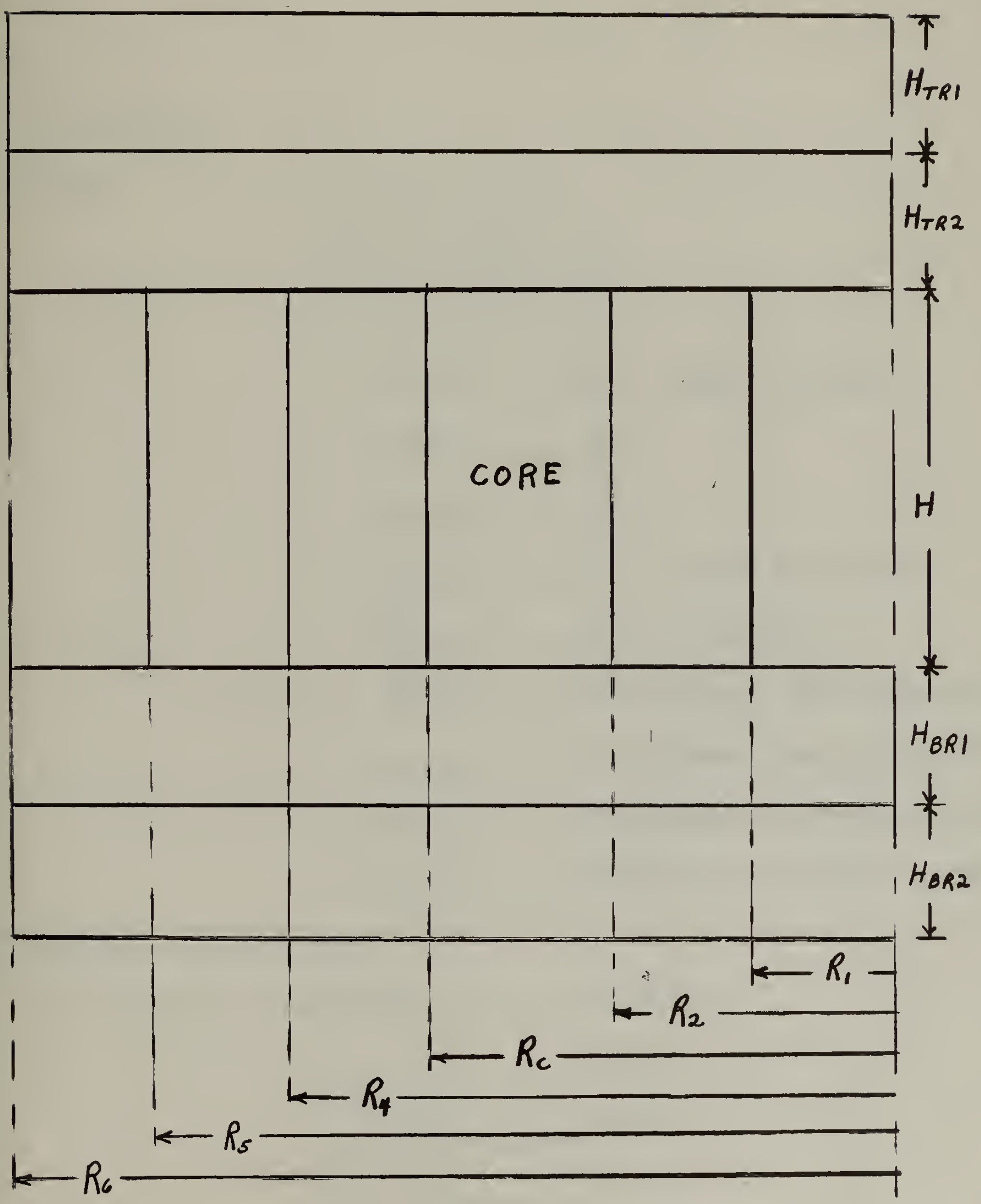


TABLE E-I

DATA TAPE PREPARATION FORM

FOURIER METHOD FOR CYLINDER

fc TAPE 404-307-39 LEE AND GARDNER

(24,6)

NOT PA

///Dimensions

DA 32/32/	+0.	
	$+1.0 \times 10^{-10}$	R_1
	$+1.0 \times 10^{-10}$	R_2
	+94.5	R_3 (core radius)
	+94.5	R_4
	+94.5	R_5
	+112.5	R_6 (outer radius)
	+243.	Core thickness
	+18.	Thickness top reflector 1
	+0.	Thickness top reflector 2
	+18.	Thickness bottom reflector 1
	+0.	Thickness bottom reflector 2

/// Radial properties

/// Thermal absorption cross sections

+0.	Σa1
+0.	Σa2
+0.	Σa3
+0.	Σa4

+ 0. $\Sigma a5$

+ .01145 $\Sigma a6$

/// k_{oo}

+ 0. k_∞ 1

+ 0. k_∞ 2

+ 0. k_∞ 3

+ 0. k_∞ 4

+ 0. k_∞ 5

+ 0. k_∞ 6

/// Thermal diffusion coefficients (non zero values)

+ 1. D_1

+ 1. D_2

+ 1. D_3

+ 1. D_4

+ 1. D_5

+ .2 D_6

/// Ages

+ 0. τ 1

+ 0. τ 2

+ 0. τ 3

+ 0. τ 4

+ 0. τ 5

+49.5 τ 6

/// Nuclear constants

+ 421. σ a (25) (barns)

+ 350. σ f (25) (barns)

+ 1.885 η (25)

+ 2.46 ν (25)

+ .910 σ_a (Al) (barns)

+ .347 σ_a (D_2O) (barns)

+ .616 x = Vol. Al/ Vol. D_2O

+ 49.5 α $\tau_c = \alpha + \beta x + \gamma x^2 + \delta x^3$

+ 0. β

+ 0. γ

+ 0. δ

+ 50.5 $\sigma_{tr}(D_2O)$

+ 4.97 σ_{tr} (Al)

+ 1. Fermi fraction of age

+ 0. Diffusion frac. of age

+ 0.

/// Longitudinal Properties

+ .01145 Σ_a Bottom reflector 2

+ .01145 Σ_a Bottom reflector 1

+ 0.

+ .01145 Σ_a Top reflector 1

+ .01145 Σ_a Top reflector 2

/// k_{oo}

+ 0. k_∞ b2

+ 0. k_∞ b1

+ 0.

+ 0. k_∞ t1

+ 0. k_∞ t2

/// Thermal diffusion coefficients

+ .2 D_{b2}

+ .2 D_{b1}

+1.

+.2 D_{t1}

+.2 D_{t2}

/// Ages

+49.5 τ B2

+49.5 τ b1

+49.5 τ t1

+49.5 τ t2

/// Initial k_∞ guesses

1.

1.

1.1 k_∞ 1

1.2 k_∞ 2

0.

0.

Note: All numbers must have + or - and decimal points.

TABLE E-II

SAMPLE DATA TAPE PRINT OUT - RUN 39

```
fc TAPE 404-306-39      LEE AND GARDNER

(24,6)

NOT PA

DA 32/32/ +0.     +1.0 x 10^-10       +1.0 x 10^-10

  +94.5   +94.5    +94.5          +112.5    +243.

  +18.    +0.      +18.            +0.

  + 0.    + 0.     + 0.            +0.       +0.       +.01145

  + 0.    + 0.     + 0.            +0.       + 0.      +0.

  + 1.    +1.      + 1.            +1.       +1.       + .2

  + 0.    +0.      + 0.            +0.       + 0.     +49.5

 + 421.  +350.      +1.885         + 2.46     + .910    +.347

   +  .616 +49.5    + 0.           + 0.       +0.      +50.5

   + 4.97  + 1.0    + 0.           + 0.

   + .01145  +.01145  +.01145        +.01145   +.01145

   +0.      +0.      +0.           +0.       +0.

    + .2     + .2     + .2           + .2      + .2

  +49.5    +49.5    +49.5         +49.5

   +1.      +1.      +1.1           +1.2      +0.       +0.

1 START AT 186
```

OUTPUTS OF POWELL CYLINDRICAL FOURIER PROGRAM

The program used in this thesis computed critical mass and infinite multiplication factor. However, the printout format of the program includes several other numbers which can be very useful in the evaluation of the answer received and also very confusing to the uninitiated.

An actual printout received for run 39 is shown below:

```
TAPE 404-306-39      LEE AND GARDNER   0541,2     4-16-57
DECIMAL
 +.13461303/-00 +.17050170/-00
 +.30481165/+06 +.30373265/+06
 -.15789275/-01 - .72544313/+02
 +.11931752/ 01 +.11916215/+01

 +.30481165/ 06 - .15789275/-01
 +.30373274/ 06 - .70792410/+03
 +.16155871/-27 +.16155871/-27 +.16155871/-27 +.68174107/+07
 +.29004734/-00 +.49500001/+02 +.49500001/+02 +.49500001/+02
 +.16155871/-27 +.16155871/-27
 +.23188179/-01 +.20502647/-01
```

Interpretation would be as follows:

In the tape number the 404 is the problem number, the 306 the programmer s number, and the 39 the run number. The numbers following the name are the time and date of the run. In each number printed, the numbers after the slant (/) locate the decimal point of the preceding number (+.11916215/+01 = 1.1916215).

The number +.16155871/-27 signifies zero. The first eight numbers are the following:

	n = 2	n = 3
Time started	Not used	Not used
Critical mass-M(25) (grams)	304811.65	303732.65
Value of critical determinant-Δ	-.015789275	-.0072544313
Infinite multiplication factor-k_∞	1.1931752	1.1916215

The next four numbers represent the approximation before the final number printed out above for the n = 2 and the n = 3 terms in the truncated Fourier series.

n = 2 (M(25))	n = 2 (Δ)
i-1 approx. = 304811.65 gm.	i-1 approx. = -.015789275
n = 3 (M(25))	n = 3 (Δ)
i-1 approx. = 303732.74 gm.	i-1 approx. = -.0007079241

The following ten numbers are read across the page from left to right and signify the following:

The first three always remain .16155871 x 10^{-27} = 0

(4) core volume -- 6817410.7 cm^3

(5) core thermal diffusion coefficient -- .29004734 cm.

(6) core age -- 49.500001 cm^2

(7) average reactor age -- 49.500001 cm^2

(8) average Fermi age -- 49.500001 cm^2

(9) average diffusion age -- .16174107 x 10^{-27} = 0

(10) This number always remains zero signified by .16174107 x 10^{-27}

The last two terms are the number of atoms per cubic centimeter x 10^{-24} of clad and moderator, respectively.

(1) clad -- .023188179 atoms/cm^3 x 10^{-24}

(2) moderator -- .020502647 atoms/cm^3 x 10^{-24}

With these outputs it is possible to compare each of the outputs with inputs to determine whether the program is operating satisfactorily. In addition by comparison of the i-1 and i approximations for each of the terms in the truncated Fourier series, it is possible to establish how fast the program is converging.

APPENDIX F

NUCLEAR DATA

A survey of the literature was undertaken to determine best values of the nuclear data that would be required. In general the best sources of information were found to be in the various Brookhaven National Laboratory reports and in the Geneva papers.

A. Cross sections

All cross sections were obtained from BNL-325 except where noted.

TABLE F-I

CROSS SECTIONS (2200 m/sec)

Nuclide	σ_a[1] (barns)	$\bar{\sigma}_a$ (barns)	$\bar{\sigma}_s$ (barns)	σ_f (barns)	$\bar{\sigma}_f$ (barns)	$N\times10^{-24}$ atoms/cm^3
H	.330	.293	38.0			.0334
H_2O	.660	.586				.0334
U-235	687.	599.	10.	580.	503.	.0474
U-238	2.75	2.44	8.3			.0474
Al	.230	.204	1.4			.06025
[2] Fe	2.53	2.24	11.			.0622
Ni	4.6	4.07	17.5			.00727
Cr	2.9	2.57	3.0			.0149
Zr	.19	.168	8.0			.0423

1. $\bar{\sigma}$ signifies effective cross section-corrected for Maxwell Boltzmann distribution.

2. Type Nr. 304 stainless steel was used.

TABLE F-II

CROSS SECTIONS (508°F)

Nuclide	$N \times 10^{-24}$ atoms/cm^3	$\bar{\sigma}_a$ [1.] barns	Σ_a cm^{-1}	σ_{tr} [2.] barns	Σ_{tr} cm^{-1}	$\bar{\sigma}_f$ barns
U-238	.0474	1.80	.0839	9.45	.440	
U-238(UO_2) [5.]	.0224	1.80	.0404	9.45	.220	
U-235	.0474	431.	20.4	96.5	4.57	358.
Fe	.0622	1.66	.1033	11.12	.694	
Ni	.00727	3.01	.0219	20.0	.146	
Cr	.0149	1.90	.0283	3.34	.050	
Stain S.			.1535		.890	
Al	.06025	.151	.0091	1.365	.0823	
H_2O [4.]	.0264	.432	.01145	62.9 [3.]	1.66	
Zr [6.]	.0423	.1245	.00526	7.95	.336	

1. $\bar{\sigma}_a = \sigma_a \frac{\sqrt{\pi}}{2} \sqrt{\frac{293.7}{537}} = .655\,\sigma_a$

2. $\sigma_{tr} = \sigma_T (1-\bar{\mu}_0)(1-4/5\frac{\sigma_a}{\sigma_T}); \quad \bar{\mu}_0 = \frac{2}{3A}$

3. Experimental-Hughes, Pile Neutron Research, page 223.

4. Density of water = 0.79 gm/cm^3 (508°F)

5. Density of UO_2 = 10.07 gm/cm^3

6. Density of Zr = 6.4 gm/cm^3

B. Fermi Age

Early measurements of Fermi Age were carried out at Oak Ridge in 1944-45, for water to metal ratios in the range from 1 to 3.5, using rod diameters of 1.18, 1.10, and .787 inches and enrichments of the order of 1%. A fission neutron source was used, and the rods were wrapped in cadmium to prevent further fissions in the lattice. The activities of cadmium covered indium foils spaced through the lattice were measured, and age to indium resonance computed by

$$\tau = \frac{1}{6} \overline{r_0}^2$$

A correction term for the difference between indium resonance and thermal was added, but this is small (about 1 cm^2).

These measurements gave a set of characteristic curves showing an age of about 40 cm^2 at a volume ratio of 3 to 1, rising sharply as the volume ratio was reduced below 1.5. An increase also occurred with reduction of the rod diameter.

Kouts, in Geneva Paper No. 600, "Exponential Experiments in Light Water", has reassessed these data in terms of the possibility of distortion through fast and epithermal fissions. These would not be prevented by the cadmium wrappings and would tend to occur more strongly at the lower volume ratios. They would have the effect of increasing the apparent age. Kouts states that, in fact, the fraction of fissions produced by neutrons above cadmium resonances (0.4 ev) at a volume ratio of 3 is about .11 while at a volume ratio of about 1.5 it rises to .58; this includes both fast fission in U-238 and

epithermal in U-235.

Kouts made measurements of M^2, the migration area, in low enrichment water lattices with volume ratios of 1 to 3. The age computations which he felt were the most accurate were based on the relationship

$$k_\infty = \eta \varepsilon p f = \exp (B^2M^2)$$

His method was that of poisoning the lattices with increasing amounts of boric acid, computing B^2 by differentiation of the thermal flux (found by measuring indium foil activities), computing f, and solving for M^2, considering the other factors to remain constant.

His results were markedly different from the Oak Ridge curves. They show a curve very slowly rising with decreasing volume ratios, and, indeed, in one case flat within the experimental error. Measurements made with 0.600 inch rods are consistently greater than those with 0.750 inch rods, but can still be considered almost flat, with a mean value of about 30 cm^2.

Chernick discusses in his paper, read at Geneva (No. 603, "The Theory of Uranium Water Lattices") methods of calculating age, and, while he does not show actual calculations or tabulate results, he does give a simplified formula

$$\tau = \frac{1}{3} \left[1 + 3\mu + e^{\mu/2} \right]$$

which does not include volume ratio as a variable and states that "Calculated values of the age of fission neutrons to indium resonance in water to uranium mixtures show little variation if the uranium to water volume ratio is less than

unity...,For equal volumes of uranium and water, calculated ages have run about 20% higher than the value for pure water."

Chernick gives as the best current value for age to indium resonance in pure water, 25 cm^2.

$$1.2 \times 25 = 30 \text{ cm}^2$$

Also included in Chernick's paper is a formula attributed to Ehrlich and Deutsch of KAPL

$$\tau = \tau_{H_2O} \frac{(1+v)^2}{(\rho+vx)^2}$$

where v = volume ratio, uranium to water

ρ = density of water

x = 0.38 for stainless steel

0.40 for Zr.

$$\tau = 25 \frac{(1+1)^2}{(1+.38)^2} = 52.5 \text{ cm}^2$$

Deutsch in Nucleonics (January 1957) gives a method for computing age in mixtures of waters and various metals such as Al, Fe, and Zr, for high enrichment reactors. These ages will be in the range of 40 to 50 or more, but they are apparently based on significant volumes of cladding material and insignificant volumes of uranium. These, then, do not fit our problem. Because of the similarity between this method and the formula ascribed to Deutsch by Chernick it seems possible that that formula might well have been intended for use with high enrichments also, and the "v" in it might have been intended to express total metal to moderator ratio, being largely made up of cladding volume.

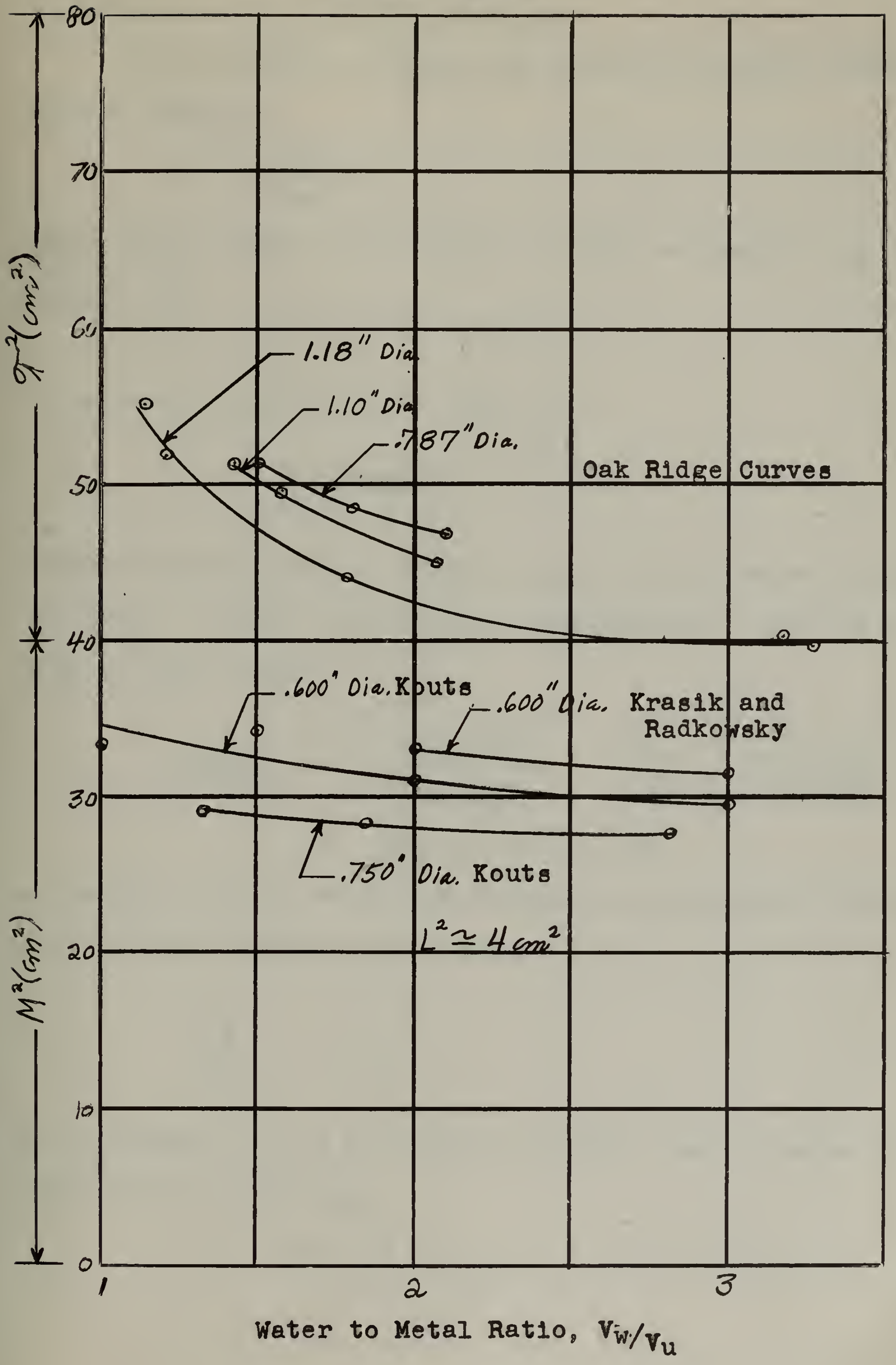
Finally, J. E. Wilkins et. al., in Geneva Paper No. 597, "Status of Experimental and Theoretical Information on Neutron Slowing Down Distributions in Hydrogenous Media", discuss computation methods for strongly absorbing media in which thermal diffusion area is small compared to age, and compares results to Kouts' data. The calculated line follows Kouts' points for 0.600 inch rods closely but turns sharply up at a volume ratio of unity to a value of almost 38, compared to Kouts' measurement of 32.5.

Since Powell's program offers a choice of using the diffusion or the Fermi formula for the age, for our purposes it appeared best to use the data of Kouts' and Chernick's theory; since Kouts' data was obtained from the physical reality by the Fermi formula, the machine computation will also be based on the Fermi formula.

The value selected was a constant value of 30 cm^2 age to indium resonance for water to uranium ratios of 1 to 3. A 1 cm^2 correction to thermal from indium resonance gives an age of 31 cm^2 for all enrichments and rod sizes.

Figure F-I

Fermi Age and Migration Area Measurements



C. Estimation of reflector savings

Theoretically, savings for an infinite water reflector, λ should approach

$$\lambda = \frac{D_c}{D_r} L_r$$

Since σ_{tr} for water is so large, it will overshadow σ_{tr} for U and SS, and

$$D_c \simeq D_r$$

At room temperature, L for water is

$$L = \frac{1}{\sqrt{3 \Sigma_a \Sigma_{tr}}} \simeq 2.9$$

However experimentally, values range from approximately six to eight. L will also vary with temperature. For the condition in this thesis

$$\frac{L\ \text{hot}}{L\ \text{cold}} = \sqrt{\frac{\Sigma_{ac} \Sigma_{trc}}{\Sigma_{aH} \Sigma_{trH}}} = \frac{N_c}{N_H} \sqrt{\frac{\sigma_{ac} \sigma_{trH}}{\sigma_{aH} \sigma_{trH}}}$$

$$= \frac{1}{.79} \times 1.35 \simeq 1.7$$

We assume that the reflector savings will vary with temperature in the same way that L does

$$\frac{\lambda_H}{\lambda_c} = 1.7$$

By selecting the end of the experimental range closest to the theoretical prediction

$$\lambda_H = 1.7\ \lambda_c = 1.7 \times 6 = 10.2 \simeq 10\ \text{cm.}$$

APPENDIX G

COMPUTATION OF CORE RADIUS AND NUMBER OF RODS

A method for determining necessary core dimensions for the transfer of predetermined thermal powers, given the unit cell dimensions, was developed from Rohsenow, Lewins, and Barger's paper, "Steady State Temperature Distributions in a Nuclear Reactor with End and Center Fed Coolant."[1] Rohsenow, Lewins, and Barger develop functions P and Q for the temperatures of the fuel centerline, the fuel clad interface, the coolant clad interface, and the coolant bulk temperature. Function P expresses, in dimensionless form, the temperature difference between the point in question and the coolant inlet; function Q expresses, also in dimensionless form, the heat transfer resistance between the area and the coolant.

It is shown in the paper that the maximum P along a channel, and hence the highest temperature, is

$$P_m^2 = Q^2 + 1$$

Since we were investigating maximum performance of power reactors, the limitations placed on our cores were principally those of temperature, and, of these, it appeared that temperature at the fuel rod centerline and of the coolant bulk temperature would be the most critical. Further, it appeared that if each of these limitations were just met at full power, that the most efficient reactor, under the prescribed conditions, had been designed.

The functions given by Rohsenow, Lewins, and Barger for

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[1] Rohsenow, Lewins, and Barger, op cit.

the end fed case are as follows:

Coolant Bulk Temperature:

$$P = \frac{\pi \Delta t_c}{q''_m} \times \frac{wc}{A} \cdot \frac{a_o}{a_f} - \sin \frac{\pi}{2} \frac{a_o}{a_f}$$

$$Q = 0$$

where Δt_c = t coolant max - t coolant inlet, °F

q''_m = heat transfer rate at midpoint of rod, $\frac{Btu}{hr\ ft^2}$

w = weight rate of coolant flow, lb./hr.

c = coolant specific heat, $\frac{Btu}{lb. ^\circ F}$

A = channel cross section area ft^2

a_o = rod heat transfer area, midpoint to end of core, ft^2

a_f = rod heat transfer area extended through reflector savings, ft^2

Centerline Temperature:

$$P = \frac{\pi \Delta t_f}{q''_m} \times \frac{wc}{A} \cdot \frac{a_o}{a_f} - \sin \frac{\pi}{2} \frac{a_o}{a_f}$$

$$Q = \pi \frac{wc}{A} \cdot \frac{a_o}{a_f} \left[\frac{1}{h} + \frac{1}{U_c} + \frac{R_c}{R_o} \times \frac{1}{\chi k_f} \times \frac{I_o(\chi_o R_o) - 1}{I_1(\chi_o R_o)} \right]$$

where h = film heat transfer coefficient

U_c = effective heat transfer coefficient for bond, clad and scale

R_o = radius of fuel region

R_c = radius to outside of cladding

$$\frac{1}{U_c} = \frac{R_c \ln(R_c/R_o)}{k_c} + \frac{1}{h_s} + \frac{1}{h_b}$$

the third term expresses the resistance in the rod, assuming that

$$\phi = A I_o(\chi_o R_o)$$

Δt_f = t of fuel centerline - t coolant inlet

In order to obtain a simultaneous solution for given

$\Delta t_f, \Delta t_c$ determined by fuel melting and coolant boiling t's

$$\left.\begin{array}{l} h,\ U_c \\ \kappa,\ k_f,\ k_c,\ c \\ R_o,\ R_c \end{array}\right\}$$ determined by materials and dimensions of the unit cell

in each case, P^2 was not equal to $Q^2 + 1$. In the equation for the coolant bulk temperature thus obtained, this leads to $P = 1$, and the resulting equation was solved for w

$$w = \frac{(1 + \sin \frac{\pi}{2} \frac{a_o}{a_f})(A\ q_m'')}{\pi\ \Delta t_c \cdot c \cdot a_o/a_f}$$

Physically, this means that we have set coolant flow sufficient to maintain the bulk temperature within the limits set.

This value of w was then substituted in the fuel centerline equation; that is, knowing the coolant flow rate, we determined the amount of heat that could be generated in the rod without exceeding the temperature limit set.

$$\frac{\pi^2\ \Delta t_f^2}{q_m''^2} \cdot \frac{(1 + \sin \frac{\pi}{2} \frac{a_o}{a_f})^2 (Aq_m'')^2}{\pi^2\ \Delta t_c^2\ c^2 (a_o/a_f)^2} \cdot \frac{c^2}{A^2} \cdot \left(\frac{a_o}{a_f}\right)^2$$

$$- \frac{2\pi\ \Delta t_f}{q_m''} \cdot \frac{(1 + \sin \frac{\pi}{2} \frac{a_o}{a_f})(Aq_m'')}{\pi\ \Delta t_c \cdot c \cdot a_o/a_f} \cdot \frac{c}{A} \cdot \frac{a_o}{a_f} \sin \frac{\pi}{2} \frac{a_o}{a_f}$$

$$+ \sin^2 \frac{\pi}{2} \frac{a_o}{a_f}$$

$$= \pi^2 \frac{c^2}{A^2}\ \frac{(1 + \sin \frac{\pi}{2} \frac{a_o}{a_f})^2 (Aq_m'')^2 .}{\pi^2\ \Delta t_c^2 .\ c^2 .\ (a_o/a_f)^2} \left(\frac{a_o}{a_f}\right)^2 \cdot\ QR^2 + 1$$

$$q_m''^2 = \frac{\left[\Delta t_f \left(1 + \sin\frac{\pi}{2}\frac{a_o}{a_f}\right) - \Delta t_c \sin\frac{\pi}{2}\frac{a_o}{a_f}\right]^2 - (\Delta t_c)^2}{\left(1 + \sin\frac{\pi}{2}\frac{a_o}{a_f}\right)^2 \times Q_R^{\,2}}$$

where $Q_R = \left[\frac{1}{h} + \frac{1}{U_c} + \frac{R_c}{R_o} \cdot \frac{1}{\mathcal{K}_o k_f} \cdot \frac{I_0(\mathcal{K} R_o) - 1}{I_1(\mathcal{K} R_o)}\right]$

For a particular series of reactors in which the relation between core height and reflector savings remains the same and in which the same temperature limitations apply, this can be expressed as

$$q_m'' = T / Q_R$$

where $$T^2 = \frac{\left[\Delta t_f\left(1+\sin\frac{\pi}{2}\frac{a_o}{a_f}\right) - \Delta t_c \sin\frac{\pi}{2}\frac{a_o}{a_f}\right]^2 - (\Delta t_c)^2}{\left(1+\sin\frac{\pi}{2}\frac{a_o}{a_f}\right)^2}$$

In the class of reactors under consideration, core heights will be of the order of 8 feet and reflector savings of the order of 10 cm.

$$\sin\left(\frac{\pi}{2}\frac{a_o}{a_f}\right) = \sin\left(\frac{\pi}{2}\cdot\frac{115}{125}\right) = \sin 83.0^\circ = 0.99 \simeq 1$$

if $\sin\frac{\pi}{2}\frac{a_o}{a_f}$ is taken as constant and equal to 1,

$$T^2 = \Delta t_f \left(\Delta t_f - \Delta t_c\right)$$

q_m'' is the heat transfer rate at the midpoint of the central rod. The heat transferred in the entire rod, q_o, is

$$q_o = q_m'' \int_{-a_o}^{+a_o} \cos\left(\frac{\pi}{2}\frac{a}{a_f}\right) da$$

$$= q_m'' \cdot \frac{4\,a_f}{\pi} \sin\left(\frac{\pi}{2}\frac{a_o}{a_f}\right) \simeq \frac{4 a_f\, q_m''}{\pi}$$

And the average power developed over all the rods, (assuming a J_o flux distribution) $\overline{q}$, is

$$\overline{q}\,\pi R^2 = \int_0^R q_o\, J_o \left(\frac{2.405r}{R'}\right) 2\pi r\, dr$$

$$\overline{q} = q_o \cdot \frac{2}{2.405}\,\frac{R'}{R}\,J_1\left(\frac{2.405R}{R'}\right)$$

where: R = core radius

$R' = R + \lambda$

λ = reflector savings

The thermal power generated in the reactor,

$$P_T = \overline{q} \times N \qquad \text{where } N = \text{no. of rods}$$

$$\frac{P_T}{N} = \overline{q} = q_o \times .83 \cdot \frac{R}{R'}\,J_1\left(\frac{2.405R}{R'}\right)$$

$$P_T = \frac{.83\pi q_o\, R(R+\lambda)}{A_u}\,J_1\left(\frac{2.405R}{R'}\right)$$

A_u = cross section area of unit cell

This may be solved for R, and the number of rods may also be found from

$$N = \pi R^2/A_u$$

Hot channel factors are engineering safety factors applied at appropriate points in the heat transfer calculations where uncertainties exist to assure a safe design. They may also be used to correct for the non-uniformity of neutron flux across the reactor, but this was accounted for in the process of averaging an assumed Bessel function J_o flux distribution across the core to obtain the average power developed in one

rod. We used only the engineering hot channel factors, therefore, and adopted a set very similar to those described by J. W. Simpson, et. al., in "Description of the Pressurized Water Reactor Power Plant at Shippingport, Pa.", Geneva Paper No. 815, which were as follows:

F_θ : an allowance for the uncertainty in the heat transfer coefficient, h, applied to reduce h.

F_Q : an allowance for the uncertainty in the allowable heat generation rate in the rods; applied to reduce q''_m .

$F_{\Delta T}$: an allowance to assure that the temperature rise stipulated is sufficient to prevent boiling, applied to reduce Δt_c.

The values of these factors are based on specified tolerances, known ranges of uncertainty, and engineering experience and judgment. The values which we adopted for our heat transfer calculations were as follows:

$F_\theta = 1.53$

$F_q = 1.08$

$F_{\Delta t} = 1.39$

By analysis similar to that shown above, the thermal fluxes required to produce burnout and local boiling can be computed and compared with q''_m

Burnout: $$\frac{{q''_{mbo}}^2}{{F_q}^2} = \frac{\Delta t_{wl} \left(\Delta t_{wl} - \frac{\Delta t_c}{F_{\Delta t}} \right.}{{F_\theta}^2 \left(\frac{1}{h} + \frac{1}{U_w} \right)^2}$$

Boiling: $$\frac{{q''_{mLB}}^2}{{F_q}^2} \qquad \frac{\Delta t_{wo} \left(\Delta t_{wo} - \frac{\Delta t_c}{F_{\Delta T}} \right.}{{F_\theta}^2 \, (1/h)^2}$$

Values for thermal conductivities and the film coefficient were selected as follows:

Material	k $^{Btu}/_{hr.ft.}$ °F
Uranium oxide, UO_2	0.95
Stainless steel, type 304	10.6
Zircalloy-2	8.15

film coefficient h = 6000 $^{Btu}/_{hr.ft.^2}$ °F

A sample calculation of core radius follows.

Sample Calculations for

Core Radius UO_2 Fuel, Stainless Steel Clad, Core No. 39

$$T^2 = t_f\,(\Delta t_f - \Delta t_c) = 4313\,(4313-101) = 18.2 \times 10^6$$

$$T = 4270$$

$$\frac{1}{h} = \frac{1.53}{6000} = 2.55 \times 10^{-4} \qquad \frac{ft^2\ hr\ ^oF}{Btu} = 2.55 \times 10^{-4}$$

$$\frac{1}{U} = \frac{R_c \ln R_c/R_o}{k_c} = \frac{(.215)\ \ln\left(\frac{.215}{.200}\right)}{12 \times 10.6} = 1.22 \times 10^{-4}$$

$$\chi_o R_o = .211$$

$$\frac{R_c}{k_f} \times \frac{1}{\chi R_o} \times \frac{I_o(\chi R_o) - 1}{I_1(\chi R_o)}$$

$$= \frac{.215}{12x.95} \times \frac{1}{.211} \times \frac{.0112}{.1061} = \underline{94.5 \times 10^{-4}}$$

$$Q_R = 98.3 \times 10^{-4}$$

$$q''_m = \frac{T}{Q_R} = \frac{4270}{1.08 \times 98.3 \times 10^{-4}} = 40.3 \times 10^4$$

Burnout Check

$$\frac{1}{h} + \frac{1}{U} = 2.55 \times 10^{-4} + 1.22x10^{-4} = 3.77 \times 10^{-4}$$

$$q''_{m_{Bo}} = \frac{2060}{\frac{1}{h} + \frac{1}{U}} = \frac{2060}{3.77x10^{-4}} = 547\ x10^4 \geq 2\ q''_m$$

Local Boiling Check

$$\frac{q''_m}{F_Q} = \frac{2.2 \times 10^6}{1.39} \geq q''_m$$

$$q''_m = 40.3 \times 10^4$$

1/2 core height, $H = 4$ ft.

Radius to outside of clad, $R_c = .215$ in.

Radius of unit cell, $R_1 = .880$ cm.

$$q_o = q_m'' \times 8R_cH = \frac{8(40.3\times10^4)(.215)(4)}{12} \times 2.93 \times 10^{-7}$$

$$= .0676 \text{ mw}$$

$$R(R+10) \times J_1\left(\frac{2.405R}{R+10}\right) = \frac{P_T \times R_1{}^2}{.83\,q_o} = 5510$$

Core radius, $R = 94.5$ cm.

Number of rods, $N = \dfrac{R^2}{R_1{}^2} = 11540$

APPENDIX H

CORRELATION OF THE POWELL FOURIER PROGRAM WITH EXPERIMENTAL DATA

A series of test runs was made with the Powell Fourier program for Whirlwind, using data from experiments, in order to compare the results from the program with experimental results. The experimental data consisted of the six lattices tested by Krasik and Radkowsky[1.] (aluminum clad uranium metal fuel in light water, with characteristics as shown in Table H-I) and the three lattices tested by J. R. Brown et al. (aluminum clad uranium oxide, UO_2, lattices also in light water), whose characteristics are also shown in Table H-I.

Since values were published for the fast fission factor and the resonance escape probability for these lattices, these values were used in the computation of the program input "$\eta \varepsilon p$". However, values were also calculated for ε and p by the methods shown in Appendix D. The largest error in the calculated "$\eta \varepsilon p$" was 1.9%, and the average about 1%. Adjusted cross sections were computed as discussed in Appendix C and in the Procedure. At first these were adjusted to an arbitrary density and atomic weight of one, and this value was sent into the computer to replace the values for aluminum which were a part of the program. Later it was found that these corrections were not being made by the

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1. Krasik and Radkowsky, op cit.
2. Brown, Nooroff, Frantz, Volpe, and Harris, op. cit.

machine, and the cross sections were then adjusted to the aluminum values, which were left undisturbed.

These runs gave results for critical mass of U-235 which were consistently too high by about 30%. In an effort to find a parameter which could be used to adjust the results to bring them closer to the experimental results, runs were made in which the input "$\eta \varepsilon p$" was varied by a series of arbitrary factors. This is the input in which the greatest uncertainty exists; it is also one which may be changed without requiring any compensating changes in other inputs. And finally, the calculations are quite sensitive to it; a change of 1% in $\eta \varepsilon p$ changes the critical mass by about 2.5%. The results of this series are shown in Table H-II and Figure H-1 in which fractional variation of $\eta \varepsilon p$ from the calculated value is plotted against the fractional variation of the resulting computed critical mass from the experimental.

It will be seen that these results form a reasonable family of curves and appear to be fairly consistent. In order to give an average value of one for M_{25} (calculated)/M_{25}(experimental), an arbitrary factor of 1.1 was chosen for $\eta \varepsilon p / \eta \varepsilon p$ (calculated). This method of correction was chosen over the alternate method of using $\eta \varepsilon p$(calculated) and correcting the resulting M_{25} by a factor of 1/1.3 on the following basis:

1. The curves were steeper at $\eta \varepsilon p / \eta \varepsilon p$ calc. = 1.1 than at $\eta \varepsilon p / \eta \varepsilon p$ calc. = 1.0. Thus an error of 1% in $\eta \varepsilon p$ would make an M_{25} error of only about 2%, against an error of 5 or 6% if $\eta \varepsilon p$ were in

the other range.

2. The spread of M_{25}calc./M_{25}exp. was narrower, about ± 8%, at $\eta\varepsilon p/\eta\varepsilon p$(calc.) = 1.1 than that at $\eta\varepsilon p/\eta\varepsilon p$ calc. = 1.0, where it was about ± 10%.

It thus appeared that both the error and the range of error would be smaller using the arbitrary factor of 1.1 in $\eta\varepsilon p$ than if an arbitrary 1/1.3 were applied to critical masses computed from $\eta\varepsilon p$ calc..

There is some measure of physical justification for this correction. In relatively tight lattices such as these, there is a considerable amount of epithermal fission, amounting to as much as 10% or more of the total fissions, according to Kouts.[1] If these did occur to a significant degree, they would tend to reduce the mass of U-235 necessary for criticality, but would not be reflected in the calculation. And if they do occur, a not unreasonable way to account for them seemed to be by a fifth factor in k_∞, which is effectively the correction which was made.

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[1] Kouts, Price, Downes, Sher, and Walsh, op. cit.

FIGURE H-1.

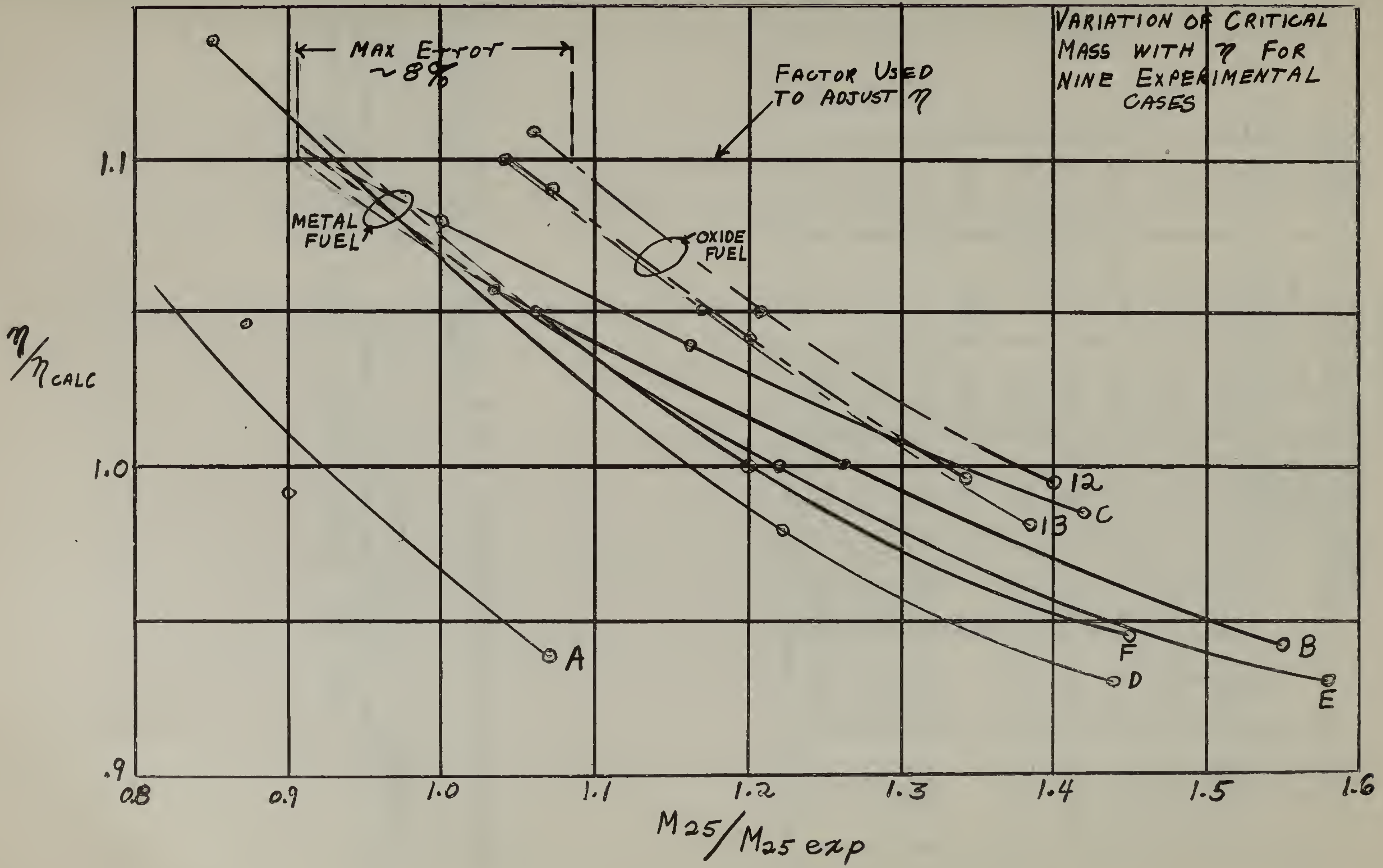

VARIATION OF CRITICAL MASS WITH η FOR NINE EXPERIMENTAL CASES
MAX ERROR ~8%
FACTOR USED TO ADJUST η
METAL FUEL
OXIDE FUEL
η/η_{CALC}
1.1
1.0
.9
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
$M_{25}/M_{25\,exp}$
A
12
C
13
F
B
D
E

TABLE H-I

Results of Critical Experiments

Core	V_1/V_o	(%) Enr.	Rod Diam.	Nr. Rods	M(25) x 10^4 gm	p	ε	f
[1] A	3	1.3	0.6"	266	1.44	.850	1.047	.849
B	2	1.3	0.6"	335	1.82	.794	1.061	.890
C	1.5	1.3	0.6"	478	2.58	.720	1.072	.918
D	3	1.15	0.6"	380	1.80	.839	1.050	.836
E	2	1.15	0.6"	463	2.22	.792	1.065	.881
F	3	1.3	.387"	631	1.42	-	-	-
[2] 12	3	1.3	0.6"	1268	2.42	.810	1.039	.875
13	4	1.3	0.6"	1025	1.98	.835	1.033	.838
14	5	1.3	0.6"	986	1.90	.870	1.029	.810

1. Cores A, B, C, D, E, and F are uranium fuel measurements from Krasik and Radkowsky.

2. Cores 12, 13, 14 are UO_2 fuel from critical measurements by Brown, Noordoff, Frantz, Volpe and Harris and f, p, ε measurements by Krantz, Smith, Klein, and Baer.

TABLE H-II

Computed Results for Varied $\eta\epsilon p$

Core	p	M(25)calc. (gm) x 10^4
A	2.05	1.11
	1.95	1.26
	1.85	1.44
	1.75	1.69
	1.65	2.05
B	1.65	2.82
	1.75	2.28
	1.85	1.91
C	1.73	2.36
	1.83	1.98
	1.93	1.71
D	1.73	2.68
	1.83	2.27
E	2.00	1.89
	1.875	2.64
	1.755	2.71
	1.625	3.51
	1.50	4.88
F	1.71	2.06
	1.81	1.71
	1.91	1.46
12	1.75	3.40
	1.85	2.93
	1.95	2.57
13	1.80	2.73
	1.90	2.37
	2.00	2.10
14	1.86	2.52
	1.96	2.21
	2.06	1.98

APPENDIX J

COMPUTATION OF THE ADDITIONAL ENRICHMENT REQUIRED TO ATTAIN SPECIFIED CORE LIFE

In the body of this thesis, a series of cores are compared which are designed to have equal power outputs; that is, they are capable of the same rate of heat generation. In order to select a true optimum, however, the designs considered must be subject to a second requirement: they must be capable of the same total amount of heat production, or "power-time" of operation. The limitation on power-time is that the reactor must remain critical throughout the period of operation.

The principal nuclear reactions affecting the reactivity of a low enrichment uranium fueled reactor are the following:

U-235 -- (n,f) FP
  '----- (n,γ) U-236

U-236 -- (n, 2β) Pu-239 -- (n,f) FP
                                   '----- (n,γ) Pu-240 -- (n,γ) Pu-241 -- (n,f) FP

Production of the fissionable isotopes Pu-239 and Pu-241 tends to increase the reactivity, and their destruction to decrease it as does destruction of U-235. Production of the neutron absorbing isotopes U-236 and Pu-240 and the neutron absorbing fission products (FP in the above expressions) tends to decrease reactivity. Since these production rates will be different in every reactor, depending on the quantities of U-235 and U-238 present, the flux at which the reactor is operated, and

the resonance absorption probability in U-238, each just critical reactor will have a different, and relatively short, allowed time of operation before ultimate loss of criticality. In general, it will be necessary to increase the initial enrichment to a value greater than that required for just reaching initial criticality in order to attain practical power-times. This appendix develops a method for finding the factor by which the minimum enrichment for initial criticality of a given reactor must be increased to attain specified power-time.

While the core life is expressed in power-time, it is useful also to define flux time as a measure of the degree of exposure of the fuel. The number of reactions which take place in a medium of given Σ_1 which is exposed to a flux ϕ for a time T is

$$\Sigma_1 \phi T$$

Since, in the practical case, flux will be varied over a period of time due to changes in power demand, the total number of reactions over such a period will be

$$\Sigma_1 \int_0^T \phi (T) \, dT$$

The quantity $\int_0^T \phi (T) \, dT$ is defined as the flux time, t.

Following the method of Spinrad, Carter, and Eggler,[1] the following nomenclature is defined:

---

[1] Spinrad, B. I., Carter, J. C., and Eggler, C., Reactivity Changes and Reactivity Lifetimes of Fixed Fuel Elements in Thermal Reactors", paper no. 835 presented at the Geneva Conference on the Peaceful Uses of Atomic Energy, 1955.

$$\Sigma_{1\,28} = N_{28}\,\sigma_{28\ \mathrm{eff}}$$

$$\Sigma_{1.25} = N_{25}\,\sigma_{25}$$

$$\Sigma_{1\,49} = N_{49}\,\sigma_{49}$$

$$\Sigma_{1\,FP} = N_{FP}\,\sigma_{FP}$$

where subscripts 25, 28, 49, and FP represent U-235, U-238, Pu-239, and pairs of fission products respectively. N, the number of atoms per unit volume of the reactor, may be a function of flux time.

ICR, the initial conversion ratio, is the ratio of production of new fissionable atoms to the destruction of U-235 at startup:

$$\mathrm{ICR} = \frac{N_{28}\,\sigma_{28\ \mathrm{eff}}}{N_{25}\,\sigma_{25}} = \frac{\Sigma_{1.28}}{\Sigma_{1.25}} + \frac{\Sigma_{1.25}}{\Sigma_{1.25}}\,\eta_{25}\,\epsilon\,P_1(1-p) \qquad (1)$$

where P_1 is the non-leakage probability during slowing down to the resonance energy. Since the effects of U-236, Pu-240, and Pu-241 are small they are neglected for simplicity.

For the low enrichment case, differential equations may be set up for the flux time changes in the macroscopic cross sections:

$$\Sigma_{1.28} \simeq \Sigma_{1.28}^{o}$$

$$\frac{d}{dt}\Sigma_{1.25} = -\sigma_{25}\,\Sigma_{1\,25}$$

$$\frac{d}{dt}\Sigma_{1.49} = \sigma_{49}\left(\Sigma_{1.28} - \Sigma_{1\,49}\right)$$

$$\frac{d}{dt}\Sigma_{1.FP} = \sigma_{FP}\left[\frac{\Sigma_{1.25}}{1+\alpha_{25}} + \frac{\Sigma_{1.49}}{1+\alpha_{49}}\right]$$

Where α is the ratio of capture to fission cross sections and the superscript "o" indicates the value at startup. Solutions of these equations are obtained as follows:

$$\Sigma_{1.25} = \Sigma_{1.25}^{0} \ e^{-\sigma_{25}t} \tag{2}$$

$$\Sigma_{1\,49} = \Sigma_{1\,28} \ (1 - e^{-\sigma_{49}t}) \tag{3}$$

$$\Sigma_{1\,FP} = \frac{\Sigma_{1.25}^{0}}{1 + \alpha_{25}} \ \frac{\sigma_{FP}}{\sigma_{25}} \ 1-e^{-\sigma_{25}t} + \frac{\Sigma_{1.28}}{1 + \alpha_{49}} \left[\sigma_{FP}t - \frac{\sigma_{FP}}{\sigma_{49}} \left(1-e^{-\ 49t}\right) \right] \tag{4}$$

Since

$$ICR = \frac{\Sigma_{1.28}}{\Sigma_{1.25}^{0}}, \qquad \Sigma_{1\,28} = ICR \times \Sigma_{1.25}^{0} \tag{5}$$

And equation (4) may be written as

$$\frac{\Sigma_{1\,FP}}{\Sigma_{1\,25}^{0}} = \frac{1}{1 + \alpha_{25}} \ \frac{\sigma_{FP}}{\sigma_{25}} \ 1-e^{-\sigma_{25}t} + \frac{ICR}{1 + \alpha_{49}} \left[\sigma_{FP}t - \frac{\sigma_{FP}}{\sigma_{49}} \left(1-e^{-\sigma_{49}t}\right) \right] \tag{6}$$

or

$$\frac{N_{FP}}{N_{25}^{0}} = \frac{1}{1 + \alpha_{25}} \ 1-e^{-\sigma_{25}t} + \frac{ICR}{1 + \alpha_{49}} \left[\sigma_{25}t - \frac{\sigma_{25}}{\sigma_{49}} \left(1-e^{-\sigma_{49}t}\right) \right] \tag{7}$$

Equation (7) expresses the total number of fission product pairs produced per atom of U-235 initially charged, at any flux time, t, and thus the total number of fissions, which is, in turn, directly proportional to the total heat produced.

The productivity, Π, is defined as the difference in rates of neutron production and loss per unit of flux time. When $\Pi = 0$, the system is just critical.

For each set of core parameters studied, the concentrations of all elements represented in the just critical, or "reference", condition may be found from the cell characteristics and the critical mass of U-235 as determined by the

machine computation. For these reference systems (denoted by the subscript "R"), productivity is zero.

$$\pi - 0 = \delta\pi = (\eta_{25}-1)\ \Sigma_{25_R} - \Sigma_{28_R} - \Sigma_{1R} - \text{Leakage}_R = 0 \qquad (8)$$

After the required flux time of operation, π must again be zero.

$$\delta\pi = (\eta_{25}-1)\ \Sigma_{25} + (\eta_{49}-1)\ \Sigma_{49} - \Sigma_{28} - \Sigma_{1}\text{-Leakage-}\Sigma_{FP}=0 \qquad (9)$$

If the enrichment of the reference reactor is increased by a factor X

$$\delta\pi = (\ _{25}-1)\ X\ \Sigma_{25_R} - \Sigma_{28_R} - \Sigma_{1R} - \text{Leakage}_R \qquad (10)$$

Subtracting equation (8) from equation (10)

$$\delta\pi = (\eta_{25}-1)\ \ \Sigma_{25_R}\ (X-1) \qquad (11)$$

Assuming that Σ_{28}, Σ_{1}, and Leakage do not change appreciably during the period of operation, equation (10) may be subtracted from equation (9):

$$(\eta_{25}-1)(\Sigma_{25}-X\ \Sigma_{25_R}) - (\eta_{49}-1)\ \Sigma_{49} - \Sigma_{FP} = -\ \delta\pi \qquad (12)$$

Σ_{FP} as expressed in equation (6) may be broken up into two parts

$$\Sigma_{FP_1} = \frac{\Sigma_{25}^{o}}{1+\alpha_{25}}\ \frac{\sigma_{FP}}{\sigma_{25}}\ (1-e^{-\sigma_{25}t}) \qquad (13)$$

$$\Sigma_{FP_2} = \frac{ICR\ \Sigma_{25}^{o}}{1+\alpha_{49}}\left[\sigma_{FP}t - \frac{\sigma_{FP}}{\sigma_{49}}\left(1-e^{-\sigma_{49}t}\right)\right] \qquad (14)$$

Substituting values from equations (2), (3), (5), (13), and

(14) into equation (12):

$$(\eta_{25}-1)(X\,\Sigma_{125_R}\,e^{-\sigma_{25}t}-X\,\Sigma_{125R})-\frac{X\,\Sigma_{125_R}}{1+\alpha_{25}}\,\frac{\sigma_{FP}}{\sigma_{25}}\,(1-e^{-\sigma_{25}t})$$

$$+\,(\eta_{49}-1)\,X\,\Sigma_{1.25_R}\;ICR(1-e^{-\sigma_{49}t})$$

$$-\,ICR\left[(\eta_{49}-1)(1-e^{-\sigma_{49}t})-\frac{1}{1+\alpha_{49}}\left\{\sigma_{FP}t-\frac{\sigma_{FP}}{\sigma_{49}}(1-e^{-\sigma_{49}t})\right\}\right]$$

$$=(\eta_{25}-1)\,\Sigma_{125_R}(X-1) \qquad (15)$$

Rearranging the terms and dropping out the common factor $\Sigma_{1.25_R}$

$$X\left[(1-e^{-\sigma_{25}t})\left(1-\eta_{25}-\frac{1}{1+\alpha_{25}}\,\frac{\sigma_{FP}}{\sigma_{25}}\right)+ICR\left\{-\frac{\sigma_{FP}t}{1+\alpha_{49}}\right.\right.$$

$$\left.\left.+(1-e^{-\sigma_{49}t})\left(\eta_{49}-1+\frac{1}{1+\alpha_{49}}\,\frac{\sigma_{FP}}{\sigma_{49}}\right)\right\}\right]$$

$$=-X(\eta_{25}-1)+(\eta_{25}-1) \qquad (16)$$

Equation (16) gives the required factor of additional enrichment as a function of flux time and Initial Conversion Ratio.

Appropriate values of the constants in equations (7) and (16) for the conditions specified for the studies are:

$\sigma_{FP} = 55$ barns

$\sigma_{25} = 431$ $\quad\alpha_{25} - .204$ $\quad\eta_{25} = 2.08$

$\sigma_{49} = 687$ $\quad\alpha_{49} = .420$ $\quad\eta_{49} = 2.03$

Substituting these values, numerical forms of equations (7) and (16) are obtained:

$$X = \frac{1.08}{1.08-1.19(1-e^{-.431t}) + \mathrm{ICR}\left[1.09(1-e^{-.687T})-.0387t\right]} \qquad (17)$$

$$\frac{N_{FP}}{N_{25}^{0}} = \frac{1}{1.204}(1-e^{-.431t}) + \mathrm{ICR}\left[.304t-.441(1-e^{-.687t})\right] \qquad (18)$$

Where t is flux time x 10^{-21}

Using these equations, X and $\frac{N_{FP}}{N_{25}^{0}}$ may be plotted against flux time for a series of ICR's, as in Figure 1.

Since both ICR and N_{FP}/N_{25}^{0} depend on X, in order to determine X from Figure J-1 an iterative process is necessary. The steps are:

1. For the specified total number of fissions and from the critical mass of U-235 for the reference reactor, compute N_{FP}/N_{25}^{0}.
2. From the unit cell geometry, enrichment, fast fission factor, resonance escape probability, and slowing down non-leakage, compute ICR as in equation (1).
3. Enter the upper curves of Figure J-1 with N_{FP}/N_{25R}^{0} and ICR_R, and find the flux time of operation needed.
4. Using this flux time and ICR_R, find X from the lower set of curves. This is the first approximation.
5. In the second approximation, enter the upper

FIGURE J-1

N_{FP}/N^{0}_{25} AND ENRICHMENT RATIO FOR FLUX TIME AND ICR

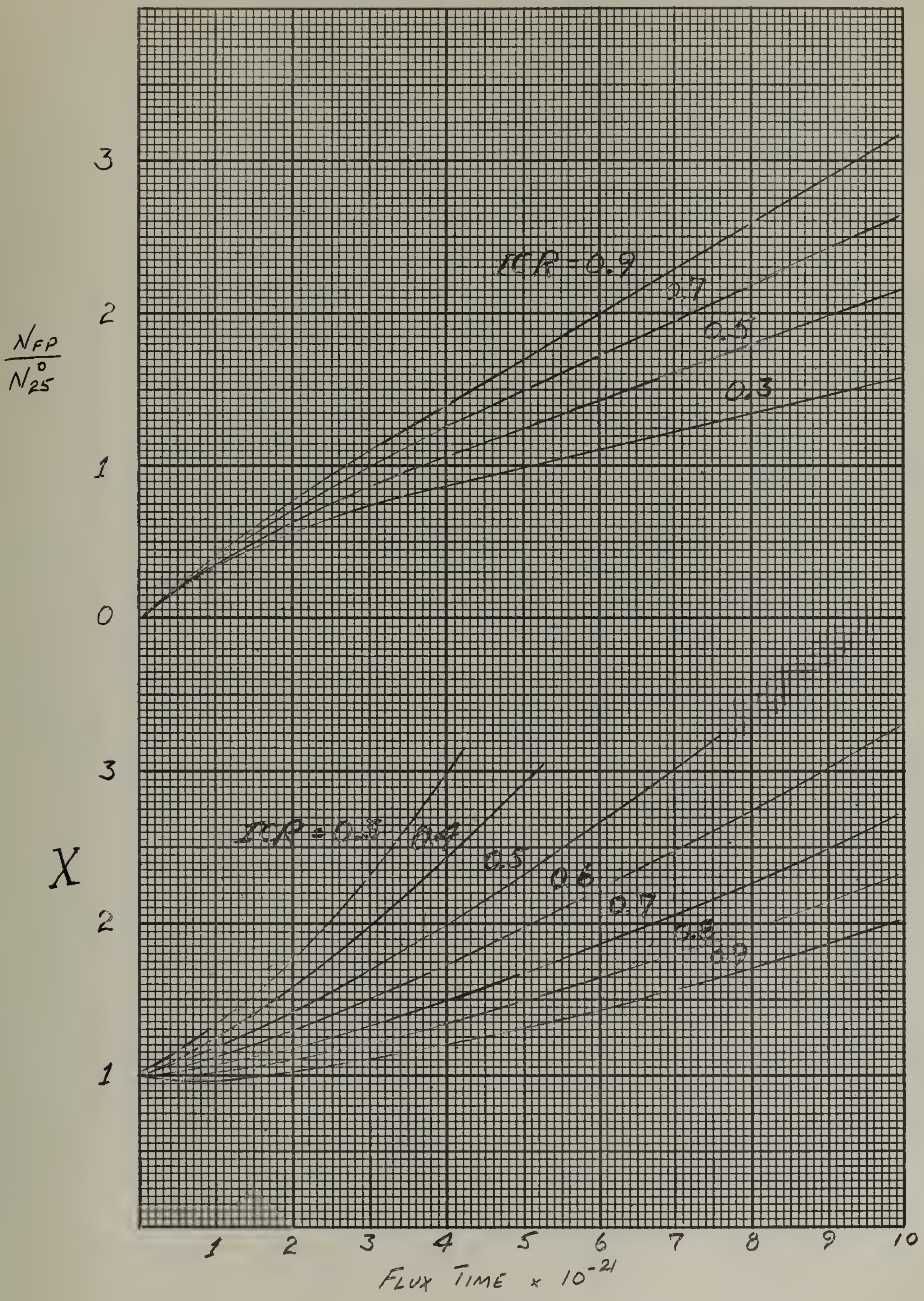
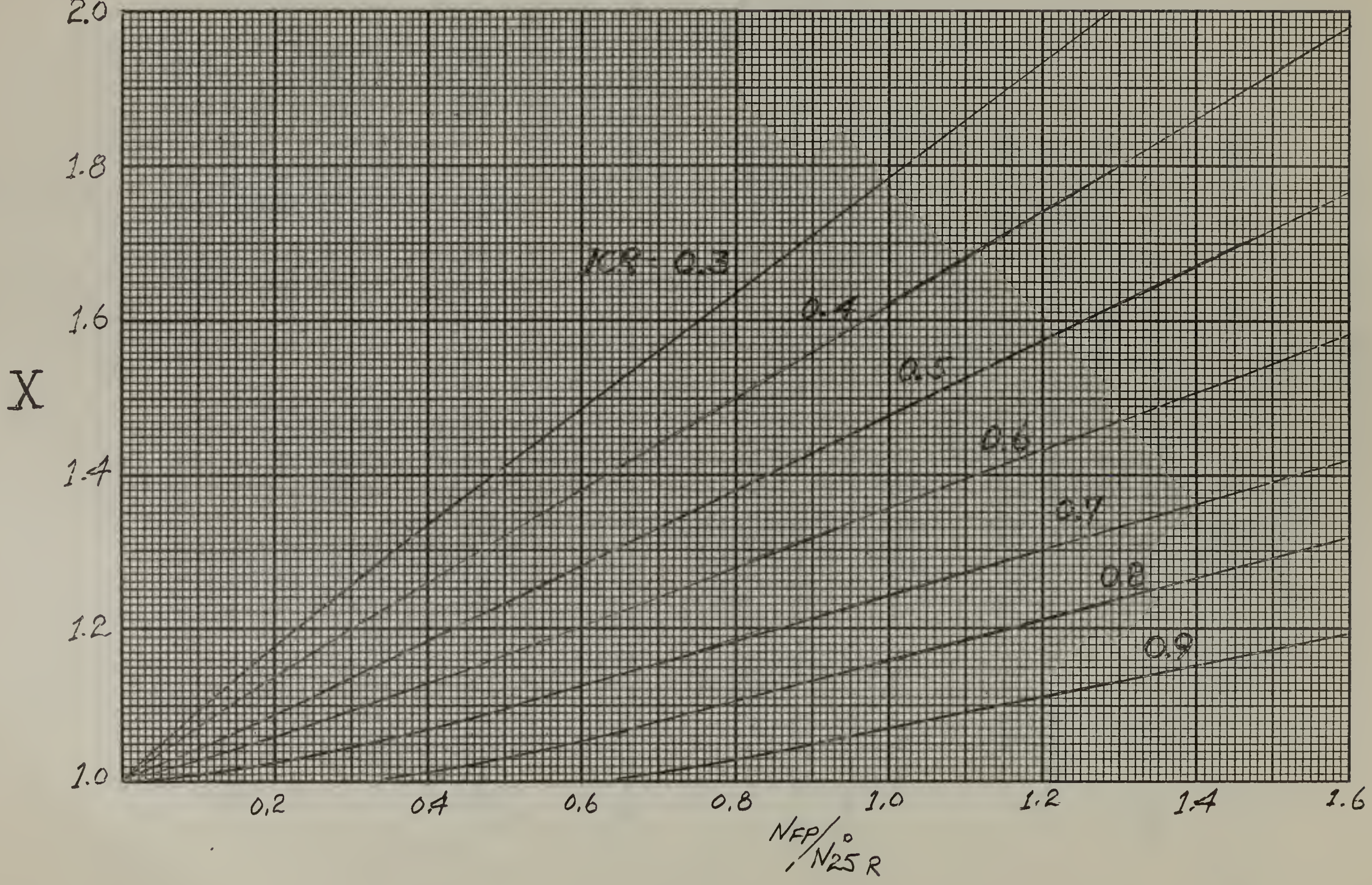


FIGURE J-2

ENRICHMENT RATIO FOR N_{FP}/N_{25}° OF THE REFERENCE REACTOR AND ICR



set of curves with

$$N_{FP}/N_{25}^{0} = 1/X(N_{FP}/N_{25R}^{0})$$

and

$$ICR \quad . = \frac{1}{X}\left(\frac{N_{28}}{N_{25}}\right)_R \frac{\sigma_{28}}{\sigma_{25}} + \eta_{25}\, \epsilon\, P_1(1-p)$$

These will give a different flux time and, from the lower curves, a new X.

6. This process is continued until the solution converges. This will usually require three or four approximations.

The solution can be somewhat shortened by replotting the data as in Figure J-2. Here flux time has been eliminated between the two curves in Figure J-1. In addition, the value of N_{FP}/N_{25}^{0} plotted for each value of X has been modified by the value of X so that N_{FP}/N_{25R}^{0} can be read directly. In finding X by Figure J-2, N_{FP}/N_{25}^{0} is found for the reference design and establishes a vertical line which needs no further modification. Along this line, ICR_R can be used to find an initial estimate of X. This X is then used to modify ICR as in step (5) above, and the process repeated until convergence. Figure J-1 is a somewhat clearer representation of the logical steps in the determination of X, but Figure J-2 will give slightly quicker convergence than Figure J-1, eliminates the modification process on N_{FP}/N_{25}^{0} , and concentrates the data in one set of curves.

APPENDIX K

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